Determine $\lim\limits_{h \to 0} \dfrac{\sqrt{a+h}-\sqrt{a}}{h\sqrt{a}}$.

Solution:

$$\lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h\sqrt{a}} = \frac{1}{\sqrt{a}} \cdot \lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h}$$

$$= \frac{1}{\sqrt{a}} \cdot \lim_{h \to 0} \frac{\sqrt{a+h}-\sqrt{a}}{h} \cdot \frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}$$

← Rationalize the numerator

$$= \frac{1}{\sqrt{a}} \cdot \lim_{h \to 0} \frac{\cancel{a}+h-\cancel{a}}{h(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a}} \cdot \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{a+h}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a}} \cdot \lim_{h \to 0} \frac{1}{\sqrt{a+h}+\sqrt{a}}$$

$$= \frac{1}{\sqrt{a}} \cdot \frac{1}{\sqrt{a+0}+\sqrt{a}}$$

$$= \frac{1}{\sqrt{a}} \cdot \frac{1}{2\sqrt{a}}$$

$$= \frac{1}{2a}$$ ✓