Prove that any odd integer, different from 1, can be written as a difference of perfect squares.

Solution: Can we use examples?
Examples can be helpful to understand a problem in more detail and this is all they can accomplish here (In some problems examples help establish a pattern but the pattern is clear here) We cannot come up with example for every case, there is infinitely many of them.
How do we cover them all? We need a <u>general</u>, all-encompassing approach.

We move away from particular numbers and choose to work with <u>algebraic expressions</u> and <u>laws of algebra</u> (always true: e.g. $a+a=2a$, $a(b+c)=ab+ac$ for all a, b, c.)
Introduce variables and form expressions relevant to the question.
Odd number can be represented as $2n+1$ or $2n-1$. Here, $2n+1$ is better. Why?
We need to prove that $2n+1 = x^2 - y^2$ for some integers x, y.
We now wish we could turn $2n+1$ into a difference of perfect squares.
Good! Wishful thinking strategy is useful here.

Can we arrange to change the form of $2n+1$ so at least one perfect square appears? Yes, we can!

$$2n+1 = (n)^2 + 2n + 1 - (n)^2$$

add / subtract

then $2n+1 = (n^2+2n+1) - n^2$

$= (n+1)^2 - (n)^2$

Then $2n+1 = x^2 - y^2$ with $x = n+1$