

Introduction to Factoring

1. GCF

The greatest common factor, GCF, or the greatest common divisor is the largest quantity (which may have both numerical and variable part) that goes into a given number of terms.

Example: $2x^2, 4x$

$$\text{gcf} = \text{gcd} = 2x$$

Exercise

Find the GCF:

$$3x^2y^2, \quad -9x^3y^4, \quad 12x^3y^2 \quad \rightarrow 3x^2y^2$$

$$21x^2, \quad 28x^4, \quad -14x \quad \rightarrow 7x$$

$$-3y^4, \quad 8y^6, \quad -6y^9 \quad \rightarrow y^4$$

$$12a^3b, \quad -16a^2b^2, \quad -24a^2b^3 \quad \rightarrow 4a^2b$$

$$16m^2n^5, \quad 24m^3n^4, \quad 32m^5n^2 \quad \rightarrow 8m^2n^2$$

2. Common Factoring

Common factoring is the process of

(a) First finding the GCF and writing it down (and opening a bracket)

(b) Then dividing the given expression by the GCF and writing the result down in brackets)

Example:

$$16x^2 - 24xy = 8x(2x - 3y)$$

Exercises:

$$1. \quad 8x^2y^2 + 6x^3y^3 - 2xy^4 = 2xy^2(4x + 3x^2y - y^2)$$

$$2. \quad (2x+4)(x-3) - 5(x-3) = (x-3)[(2x+4) - 5] = (x-3)(2x+4-5) = (x-3)(2x-1)$$

$$3. \quad y(y-6) + 9(y-6) - 3y^2(y-6) = (y-6)(y+9-3y^2)$$

$$4. \quad (z+2)^2 - 5(z+2) = (z+2)(z+2) - 5(z+2) = (z+2)(z+2-5) = (z+2)(z-3)$$

$$5. \quad (x+1)(x^2-2x+3) - 5(x-3)(x^2-2x+3) = (x^2-2x+3)[(x+1) - 5(x-3)] \\ = (x^2-2x+3)[x+1-5x+15] = (x^2-2x+3)(16-4x) \\ = 4(x^2-2x+3)(4-x)$$

Factor (fully) each of the following:

1. a) $4x+4y$ b) $5c+5d$ c) $7a-7b$ d) $3x-3y$
 e) $ax+az$ f) $ca+cb$ g) $bx-by$ h) $da-db$
 i) $5x+5y+5z$ j) $3a-3b-3c$ k) $ax+ay+az$ l) $cx-dx-dx$
2. a) $3x+6$ b) $8y+32$ c) $2y-16$ d) $8x-40$
 e) $6n+9$ f) $10x-15$ g) $24x+8$ h) $20+12a$
3. a) x^2+xy b) a^3-ab c) x^3-x^2y d) x^4-x^2y
 e) a^3x+a^2y f) a^3+a^2+2a g) $a^2b^2+ab^3$ h) $x^4y^2-x^2y^4$
4. a) $am+mx-2m$ b) $ab+a$ c) $xy+y$ d) $cd-c$
 e) $xy-xz-x$ f) $3xy-3xz-3x$ g) $8x+16y+24z$ h) $15a-12b-6c$
5. a) $15x+10y+5z$ b) $ax+ay-az$ c) x^3+2x^2-5x d) $4x^3-8x^2+2x$
 e) $2ax+4ay-6az$ f) $2xy-4x^2y^2-6xyz$ g) $2x^4-6x^3+12x^2$ h) $16a^5-6a^4+3a^3$
6. a) $4j+12p-18p+22r$ b) $15t^3-6t^2+30t+3$ c) $51p^4-17p^3$
 d) $42gh^3-7g^2h^2+10gh^4$ e) $8m-8my$ f) $14xy+63xy^2-35x^2y$
7. a) $2x(x+5)+3(x+5)$ b) $3x(x+y)-2y(x+y)$ c) $4x^2(x^2-5)+11(x^2-5)$
 d) $5x(7x-8)-6(7x-8)$ e) $x^2(2x-1)+3x(2x-1)-4(2x-1)$
 f) $5x^2(2x^2+x+1)-3(2x^2+x+1)+5(2x^2+x+1)$ g) $3x(1-7x)-(1-7x)$
 h) $x^2(x-1)+x(x-1)-(x-1)$ i) $x(x^2-xy+y^2)-(x^2-xy+y^2)$
8. a) $2x(x^2-3x-7)-4(x^2-3x-7)$ b) $7x^5(x+2)+14x^4(x+2)$
 c) $3x^2y(xy-8)+6xy^2(xy-8)$ d) $5x(x+7)^2-3(x+7)^2$
 e) $4x^2(x-3)+8x(x-3)$ f) $2x^2y(5x-3y)-4xy(5x-3y)+6y^2(5x-3y)$
 g) $5x^3(x+5)^4+5x^2(x+5)^4$ h) $15x^4y(x^2+y^2)-20xy^4(x^2+y^2)$
9. a) $7(x+5)^2-8(x+5)$ b) $8(x+4)^5-3(x+4)^4$
 c) $10(x+7)^4-20(x+7)^3$ d) $10x(x+7)^4-20(x+7)^5$
 e) $12x^4(x+3)^3+15x^5(x+3)^2$ f) $y^5(2-y^2)^3+y^3(2-y^2)^4$

Answers:

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8. a) $2(x^2-3x-7)(x-2)$ b) $7x^4(x+2)^2$ c) $3xy(xy-8)(x+2y)$
 d) $(x+7)^2(5x-3)$ e) $4x(x-3)(x+2)$ f) $2y(5x-3y)(x^2-2x+3y)$
 g) $5x^2(x+5)^4(x+1)$ h) $5xy(x^2+y^2)(3x^3-4y^3)$
 9. a) $(x+5)(7x+27)$ b) $(x+4)^4(8x+29)$ c) $10(x+7)^3(x-13)$
 d) $10(x+7)^4(-x-14)$ e) $9x^4(3x+1)$ f) $2y^3(2-y^2)^3$