MHF 4U

Investigating Oblique Asymptotes

Date:

Analyse and Sketch: $f(x)=\dfrac{x^2-3x+6}{x-2}$

i) Domain and Vertical Asymptote(s):

$x=2$

$D:\{x\in\mathbb{R} \mid x\neq 2\}$

ii) Horizontal Asymptote(s):

No HA.

why? $f(x)=\dfrac{1-\frac{3}{x}+\frac{6}{x^2}}{\frac{1}{x}-\frac{2}{x^2}}$

$\frac{1}{0}$!

iii) Intercepts:

y-int $=-3$

x-int

$0=x^2-3x+6$

No x-int.

iv) Oblique asymptote

For rational functions $f(x)=\dfrac{h(x)}{g(x)}$, where the degree of $h(x)$ is n and the degree of $g(x)$ is m, if $\boxed{n>m,}$ and $\boxed{n=m+1}$ then there is an oblique asymptote. An oblique asymptote is a line that the graph approaches that is neither vertical nor horizontal. The equation of the oblique asymptote can be determined by dividing $h(x)$ by $g(x)$.

OA!

$$\begin{array}{r|l} & x-1 \\ \hline x-2 & x^2-3x+6 \\ & -(x^2-2x) \\ \hline & -x+6 \\ & -(-x+2) \\ \hline & 4 \end{array}$$

$$\frac{x^2-3x+6}{x-2}=\frac{(x-2)(x-1)}{x-2}+\frac{4}{x-2}$$

$$f(x)=\frac{x^2-3x+6}{x-2}=x-1+\frac{4}{x-2}$$

OA!

$\therefore$ $y=x-1$ is the O.A.

Just as with horizontal asymptotes, it is possible for a function to intersect its oblique asymptote. In the space below, determine the intersection point of $f(x)=\dfrac{x^2-3x+6}{x-2}$ and the oblique asymptote.

$$\frac{x^2-3x+6}{x-2}=x-1$$

$$x^2-3x+6=(x-1)(x-2)$$

$$x^2-3x+6=x^2-3x+2$$

$$6=2$$

$$0=-4$$

$\therefore$ No crossover!

Note: A rational function can not have both a HA and an OA.

MHF 4U

Investigating Oblique Asymptotes

Date:

Analyse and Sketch: $f(x)=\frac{x^2-3x+6}{x-2}$

i) Domain and Vertical Asymptote(s):

$x=2$

$D: \{x \in \mathbb{R} \mid x \neq 2\}$

ii) Horizontal Asymptote(s):

No HA.

why? $f(x)=\frac{1-\frac{3}{x}+\frac{6}{x^2}}{\frac{1}{x}-\frac{2}{x^2}}$

$\frac{1}{0}$!

iii) Intercepts:

y-int $=-3$

x-int

$0=x^2-3x+6$

No x-int.

iv) Oblique asymptote

For rational functions $f(x)=\frac{h(x)}{g(x)}$, where the degree of $h(x)$ is n and the degree of $g(x)$ is m, $\boxed{\text{if } n>m,}$ and $\boxed{n=m+1}$ then there is an oblique asymptote. An oblique asymptote is a line that the graph approaches that is neither vertical nor horizontal. The equation of the oblique asymptote can be determined by dividing $h(x)$ by $g(x)$.

OA!

$$\begin{array}{r} x-1 \\ x-2 \enclose{longdiv}{x^2-3x+6} \\ -\ \ x^2-2x \\ \hline -x+6 \\ -(-x+2) \\ \hline 4 \end{array}$$

$$\frac{x^2-3x+6}{x-2}=\frac{(x-2)(x-1)}{x-2}+\frac{4}{x-2}$$

$$f(x)=\frac{x^2-3x+6}{x-2}=x-1+\frac{4}{x-2}$$

OA!

$\therefore\ y=x-1$ is the O.A.

Just as with horizontal asymptotes, it is possible for a function to intersect its oblique asymptote. In the space below, determine the intersection point of $f(x)=\frac{x^2-3x+6}{x-2}$ and the oblique asymptote.

$$\frac{x^2-3x+6}{x-2}=x-1$$

$$x^2-3x+6=(x-1)(x-2)$$

$$x^2-3x+6=x^2-3x+2$$

$$6=2$$

$$0=-4$$

$\therefore$ No crossover!

Note: A rational function can not have both a HA and an OA.

Investigating Oblique Asymptotes

Date:

v) Behaviour near asymptotes:

x	$f(x)=\dfrac{x^2-3x+6}{x-2}$	$y=x-1$ (oblique asymptote)	
(as $x\to\infty$) $+10000$	$9999.0004 >$	9999	As $x\to\infty$, $f(x)\to x-1$ from above
as $x\to 2^-$ 1.999	$y\to-\infty$ large neg		As $x\to 2^-$, $y\to-\infty$
2	undefined		As $x\to 2^+$, $y\to+\infty$
as $x\to 2^+$ 2.0001	$y\to+\infty$ large pos.		
-10000 (as $x\to-\infty$)	$-10001.0004 <$	-10001	As $x\to-\infty$, $f(x)\to x-1$ from below

vi) Sketch

Include the asymptotes as dotted lines.