More Rational Functions

Date: March 11/16

Rational functions are not limited to reciprocals of linear or quadratic functions. Recall that the definition of a rational function is $f(x)=\frac{h(x)}{g(x)}$, where $h(x)$ and $g(x)$ are polynomial functions.

Example 1: Analyse and sketch $f(x)=\frac{x+1}{x^2-4}$

i) Domain and Vertical Asymptote(s):

$\{x\in\mathbb{R}, x\neq\pm2\}$

VA $x=2$
$x=-2$

ii) Horizontal Asymptote(s):

$y=0$
(since degree numerator < degree denominator)

iii) Intercepts:

y-int $= -\frac{1}{4}$

x-int $= -1$

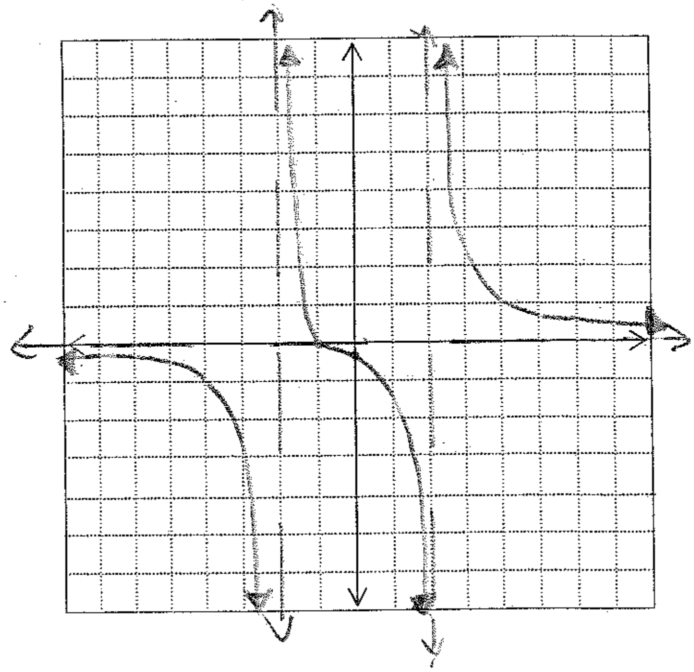
iv) Behaviour near asymptotes:

as $x\to -2^-$, $y\to -\infty$
as $x\to -2^+$, $y\to +\infty$

as $x\to 2^-$, $y\to -\infty$
as $x\to 2^+$, $y\to +\infty$

as $x\to -\infty$, $y\to 0$ from below
as $x\to +\infty$, $y\to 0$ from above

v) Sketch



Notes:

- middle looks "cubic".
- y-int is the division of the constants.

* has a "crossover" point on the HA.
(HA of $y=0$, but an x-intercept) point of intersection *

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Example 2. Analyse and sketch $f(x)=\dfrac{x+5}{x^2-3x-10}$ $=\dfrac{x+5}{(x-5)(x+2)}$

i) Domain and Vertical Asymptote(s):

VA: $x=5$

$x=-2$

ii) Horizontal Asymptote(s):

$y=0$

iii) Intercepts:

$x=-5$

$y=-\frac{1}{2}$

iv) Behaviour near asymptotes:

as $x\to -2^-$, $y\to \infty$

as $x\to -2^+$, $y\to -\infty$

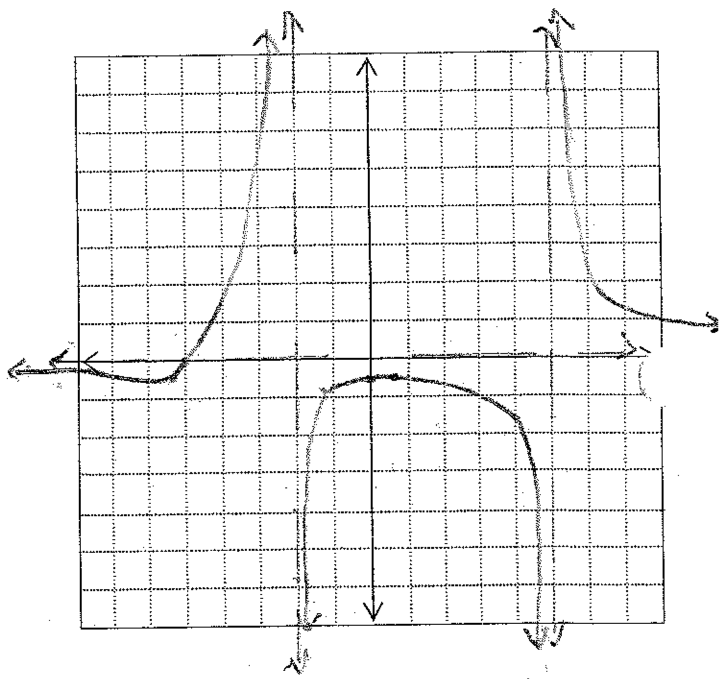
as $x\to 5^-$, $y\to -\infty$

as $x\to 5^+$, $y\to \infty$

as $x\to -\infty$, $y\to 0$ from below

as $x\to +\infty$, $y\to 0$ from above

v) Sketch



Notes: → max point is <u>not</u> ~~halfway between~~ the VAs.

→ another crossover!

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Example 3. Analyse and sketch $f(x)=\frac{x-5}{x^2-3x-10} = \frac{x-5}{(x-5)(x+2)} = \frac{1}{x+2}$ $x \neq 5, -2$

hole @ $x=5$ and $y=\frac{1}{7}$

i) Domain and Vertical Asymptote(s):

VA : $x=-2$

Note: do not say $x=5$ is a VA too!

ii) Horizontal Asymptote(s):

$y=0$

iii) Intercepts:

x-int = None

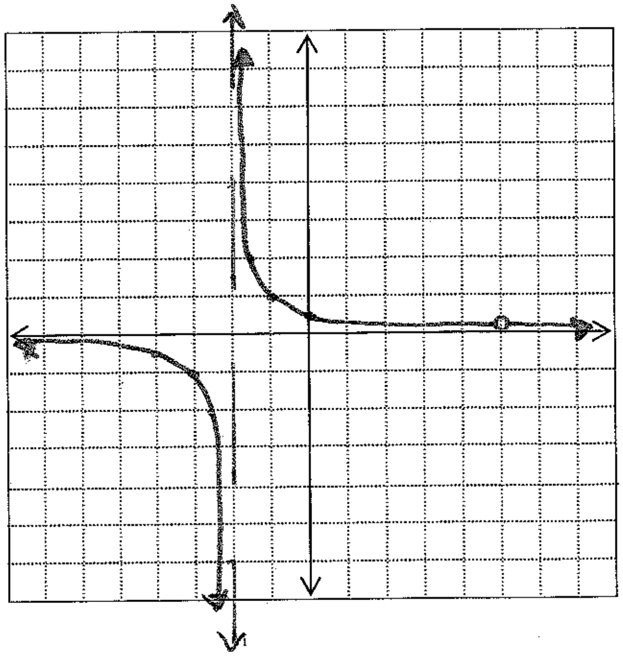
y-int = $\frac{1}{2}$

iv) Behaviour near asymptotes:

as $x \to -2^-$, $y \to -\infty$
as $x \to -2^+$ $y \to \infty$

as $x \to -\infty$, $y \to 0$ from below
as $x \to \infty$, $y \to 0$ from above

v) Sketch



Notes: When the numerator & denominator share a factor The value becomes a hole, not a V.A. (point discontinuity)

If $f(x)=\frac{h(x)}{g(x)}$, then

① If $h(a)=0$ & $g(a)=0$, then there will be a hole @ $x=a$.

② If $h(a)\neq 0$ & $g(a)=0$, then there will be a VA @ $x=a$.