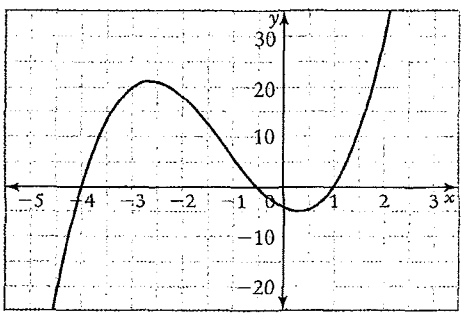
Practise

For help with questions 1 and 2, refer to Example 1.

1. For each polynomial function:
   i) state the degree and the sign of the leading coefficient
   ii) describe the end behaviour of the graph of the function
   iii) determine the x-intercepts
   a) $f(x) = (x - 4)(x + 3)(2x - 1)$
   b) $g(x) = -2(x + 2)(x - 2)(1 + x)(x - 1)$
   c) $h(x) = (3x + 2)^2(x - 4)(x + 1)(2x - 3)$
   d) $p(x) = -(x + 5)^3(x - 5)^3$

2. For each graph, do the following.
   i) State the x-intercepts.
   ii) State the intervals where the function is positive and the intervals where it is negative.
   iii) Explain whether the graph might represent a polynomial function that has zeros of order 2 or of order 3.

a)



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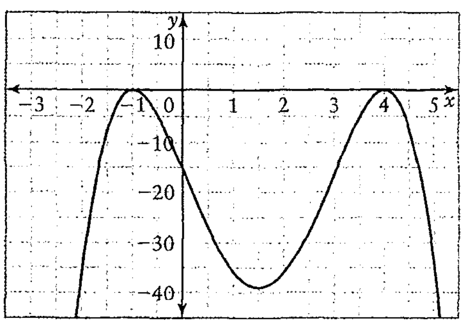

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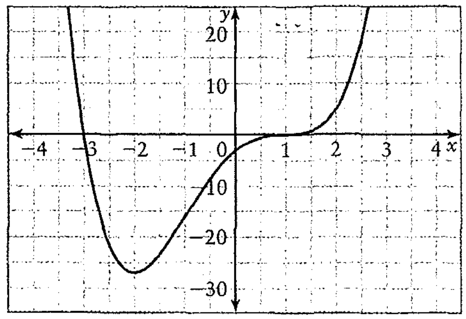

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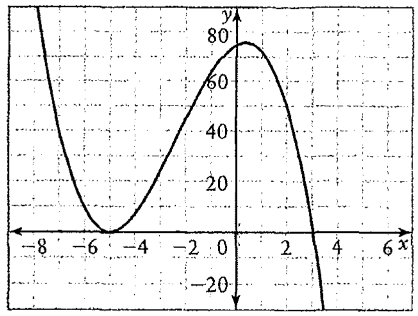
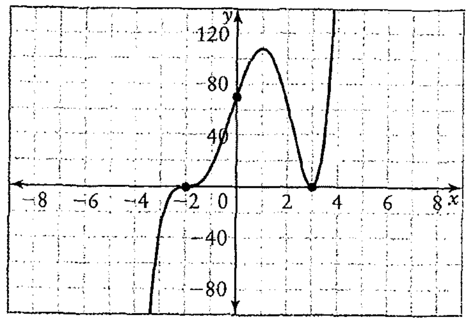
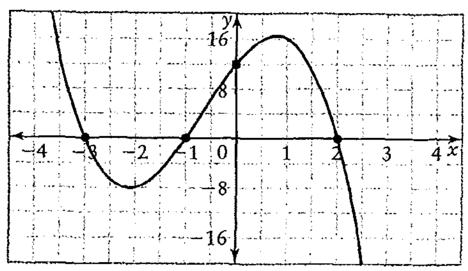


6. Determine an equation for the polynomial function that corresponds to each graph.

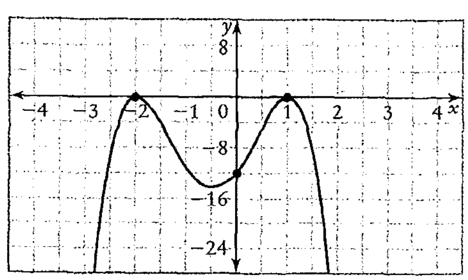
a)

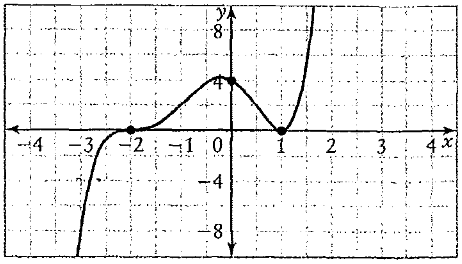


b)

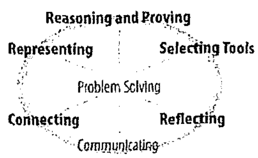


c)



d) 

9. Each polynomial function has zeros at −3, −1, 2. Write an equation for each function. Then, sketch a graph of the function.



a) a cubic function with a positive leading coefficient

b) a quartic function that touches the x-axis at -1

c) a quartic function that extends from quadrant 3 to quadrant 4

d) a quintic function that extends from quadrant 3 to quadrant 1

10. **Chapter Problem** An engineer designs a rollercoaster so that a section of the ride can be modelled by the function $h(x) = -0.000\,000\,4x(x - 15)(x - 25)(x - 45)^2(x - 60)$, where x is the horizontal distance from the boarding platform, in metres; $x \in [0, 60]$; and h is the height, in metres, above or below the boarding platform.

a) What are the similarities and differences between this polynomial function and those studied in Sections 1.1 and 1.2?

b) What useful information does this form of the equation provide that can be used to sketch a graph of the path of the rollercoaster?

c) Use the information from part b) to sketch a graph of this section of the rollercoaster.

d) Estimate the maximum and the minimum height of the rollercoaster relative to the boarding platform.

11. a) Determine the zeros of $f(x) = (2x^2 - x - 1)(x^2 - 3x - 4)$.

b) Use graphing technology to verify your answer.

13. **Use Technology** Consider the polynomial function $f(x) = (x - 3)(x - 1)(x + 2)^2 + c$, where c is a constant. Determine a value of c such that the graph of the function has each number of x-intercepts. Justify your answer graphically.

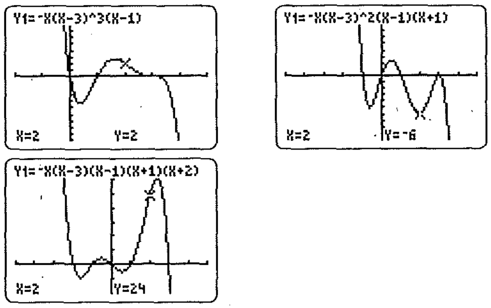
a) four

b) three

c) two

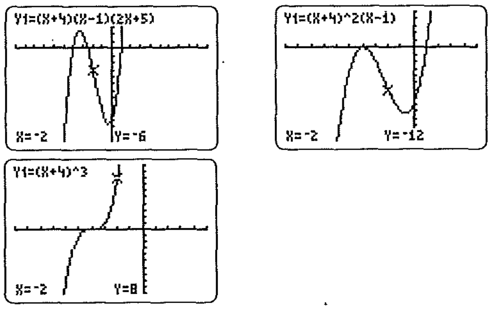
d) one

e) zero



17. a) i) cubic **ii)** cubic **iii)** cubic

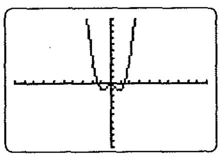
b)



c) Answers may vary. Sample answer: The number of x-intercepts equals the number of roots of the equation.

18. a) i) $S(r) = 8\pi r^2$ **ii)** $V(r) = 3\pi r^3$ **b)** Answers may vary. Sample answer: $S(r)$ quadratic, has one x-intercept, $\{x \in \mathbb{R}, x \geq 0\}$, $\{y \in \mathbb{R}, y \geq 0\}$, from quadrant 2 to quadrant 1; $V(r)$ cubic, one x-intercept, $\{x \in \mathbb{R}\}$, $\{V \in \mathbb{R}\}$ from quadrant 3 to quadrant 1

19. a)



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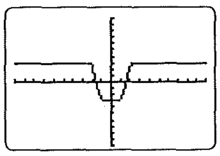
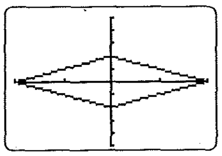


c)



1.3 Equations and Graphs of Polynomial Functions, pages 39–41

1. a) i) 3, + **ii)** quadrant 3 to quadrant 1 **iii)** 4, −3, $\frac{1}{2}$

b) i) 4, − **ii)** quadrant 3 to quadrant 4 **iii)** −2, 2, 1, −1

c) i) 5, + **ii)** quadrant 3 to quadrant 1 **iii)** $-\frac{2}{3}$, 4, −1, $\frac{3}{2}$

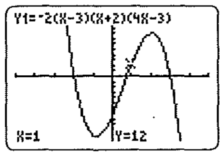
d) i) 6, − **ii)** quadrant 3 to quadrant 4 **iii)** −5, 5

2. a) i) $-4, -\frac{1}{2}, 1$ **ii)** positive, $-4 < x < -\frac{1}{2}$, $x > 1$; negative $x < -4$, $-\frac{1}{2} < x < 1$ **iii)** no zeros of order 2 or 3

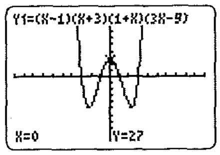
b) i) −1, 4 **ii)** negative $x < -1$, $-1 < x < 4$, $x > 4$ **iii)** could have zeros of order 2 **c) i)** −3, 1 **ii)** positive $x < -3$, $x > 1$; negative $-3 < x < 1$ **iii)** could have zeros of order 3 **d) i)** −5, 3 **ii)** positive $x < -5$, $-5 < x < 3$; negative $x > 3$ **iii)** could have zeros of order 2

3. a) i) −2, 3, $\frac{3}{4}$, all order 1 **ii)** −3, −1, 1, 3, all order 1 **iii)** order 2: −4, 1; order 1: −2, $\frac{3}{2}$ **iv)** order 3: $\frac{2}{3}$; order 2: 5; order 1: −6 **b)** graph in part ii) is even; others are neither

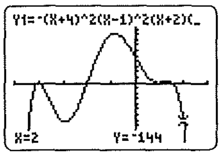
c) i)



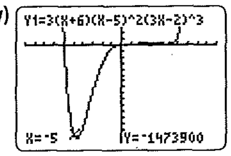
ii)



iii)



iv)

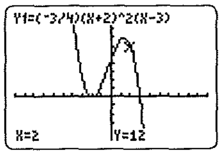


4. b) d) line, even; these functions have line symmetry about the y-axis because they are even functions. **a) c)** neither, neither; there is no symmetry about the origin or about the y-axis because these functions are neither even nor odd.

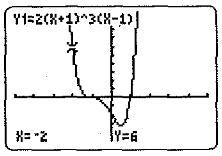
5. a) i) even **ii)** line **b) i)** odd **ii)** point **c) i)** neither **ii)** neither **d) i)** neither **ii)** neither **e) i)** even **ii)** line

6. a) $y = (x + 2)^3(x - 3)^2$ **b)** $y = -2(x + 3)(x + 1)(x - 2)$ **c)** $y = -3(x + 2)^2(x - 1)^2$ **d)** $y = 0.5(x + 2)^3(x - 1)^2$

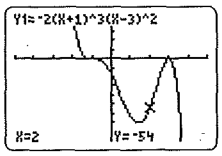
7. a) $y = -\frac{3}{4}(x + 2)^2(x - 3)$, neither



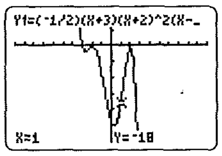
b) $y = 2(x + 1)^3(x - 1)$, neither



c) $y = -2(x + 1)^3(x - 3)^2$, neither



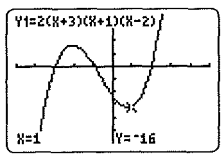
d) $y = -\frac{1}{2}(x + 3)(x + 2)^2(x - 2)^2$, neither



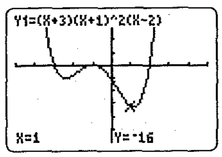
8. a) point **b)** line **c)** point **d)** point

9. Answers may vary. Sample answers:

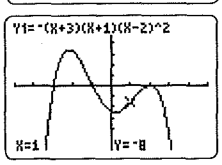
a)



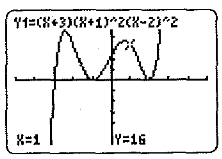
b)



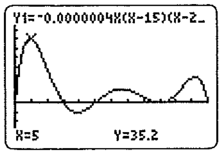
c)



d)

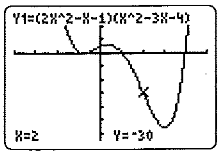


10. a) Answers may vary. **b)** Answers may vary. Sample answer: The equation provides information about the x-intercepts, the degree of the function, the sign of the leading coefficient, and the order of the zeros.

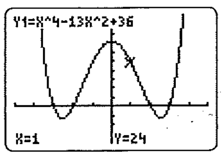
c) 

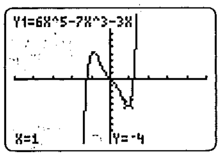
d) The maximum height is approximately 35.3 m above the platform. The minimum height is approximately 5.1 m below the platform.

11. a) $4, 1, -1, -\frac{1}{2}$

b) 

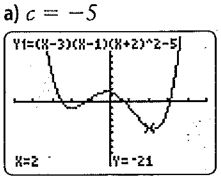
12. a) i) $3, 2, -2, -3$ **ii)** $0, -\frac{\sqrt{6}}{2}, \frac{\sqrt{6}}{2}$ **b) i)** even **ii)** odd

c) i) 

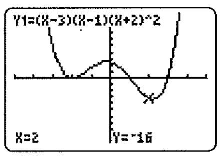
ii) 

13. Answers may vary. Sample answers:

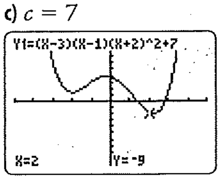
a) $c = -5$



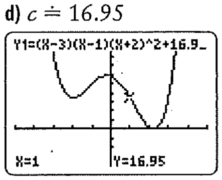
b) $c = 0$



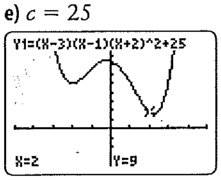
c) $c = 7$



d) $c \doteq 16.95$



e) $c = 25$



14. a) Answers may vary. Sample answer:
$f(x) = (3x - 2)(3x + 2)(x - 5)(x + 5)$,
$g(x) = 2(3x - 2)(3x + 2)(x - 5)(x + 5)$
b) $y = -3.2(3x - 2)(x - 5)$ **c)** $y = 3.2(3x - 2)(x - 5)$

15. Answers may vary. Sample answer: shifts from the origin, and an odd function must go through the origin.

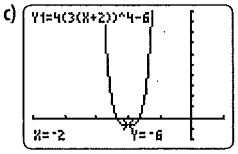
16. a) 0.35 **b)** 2.517

1.4 Transformations, pages 49–52

1. a) $a = 4$ (vertical stretch by a factor of 4), $k = 3$ (horizontal compression by a factor of $\frac{1}{3}$), $d = -2$ (translation 2 units left), $c = -6$ (translation 6 units down)

b)

$y = x^4$	$y = (3x)^4$	$y = 4(3x)^4$	$y = 4[3(x + 2)]^4 - 6$
(−2, 16)	(−2, 1296)	(−2, 5184)	(−2, −6)
(−1, 1)	(−1, 81)	(−1, 324)	(−1, 318)
(0, 0)	(0, 0)	(0, 0)	(0, 5178)
(1, 1)	(1, 81)	(1, 324)	(1, 26 238)
(2, 16)	(2, 1296)	(2, 5184)	(2, 82 938)

c) 

d) $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -6\}$, $(-2, -6)$, $x = -2$

2. a) ii **b)** iv **c)** iii **d)** i

3. a) iii **b)** iv **c)** ii **d)** i

4. a) $k =$ (3 horizontal compression by a factor of $\frac{1}{3}$), $c = -1$ (vertical translation 1 unit down), $n = 3$
b) $a = 0.4$ (vertical compression by a factor of 0.4), $d = -2$ (horizontal translation of 2 units left), $n = 2$ **c)** $c = 5$ (vertical translation of 5 units up), $n = 3$ **d)** $a = \frac{3}{4}$ (vertical compression by a factor of $\frac{3}{4}$), $k = -1$ (reflection in the y-axis), $d = 4$ (horizontal translation 4 units right), $c = 1$ (vertical translation 1 unit up), $n = 3$ **e)** $a = 2$ (vertical stretch by a factor of 2), $k = \frac{1}{3}$ (horizontal stretch by a factor of 3), $c = -5$ (vertical translation 5 units down), $n = 4$ **f)** $a = 8$ (vertical stretch by a factor of 8), $k = 2$ (horizontal compression by a factor of $\frac{1}{2}$), $c = 24$ (vertical translation 24 units up), $n = 3$

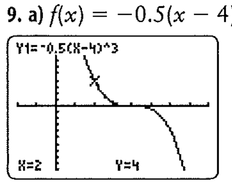
5. a) ii **b)** iv **c)** i **d)** iii

6. a) 2 units left, 1 unit down, $y = (x + 2)^2 - 1$
b) 4 units right, 5 units up, $y = (x - 4)^3 + 5$

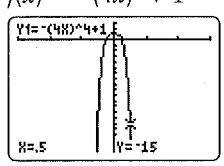
7. a) $a = -3$, $k = \frac{1}{2}$, $d = -4$, $c = 1$
b) a: vertical stretch by a factor of 3 and a reflection in the x-axis; k: horizontal stretch by a factor of 2; d: 4 units left; c: 1 unit up **c)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \leq 1\}$, $(-4, 1)$, $x = -4$
d) vertical stretch, horizontal stretch, left, up; horizontal stretch, vertical stretch, up, left

8. a) vertical compression by a factor of 0.5 and a reflection in the x-axis, translation 4 units right; $f(x) = -0.5(x - 4)^3$
b) reflection in x-axis, horizontal compression by a factor of $\frac{1}{4}$, translation 1 unit up; $f(x) = -(4x)^4 + 1$ **c)** vertical stretch by a factor of 2, horizontal stretch by a factor of 3, translation 5 units right, translation 2 units down;
$f(x) = 2\left[\frac{1}{3}(x - 5)\right]^3 - 2$

9. a) $f(x) = -0.5(x - 4)^3$



$f(x) = -(4x)^4 + 1$



$f(x) = 2\left[\frac{1}{3}(x - 5)\right]^3 - 2$