Simplifying Algebraic Expressions

Algebraic expressions contain both numbers and letters. The letters are often called **variables.**

Algebraic expressions can only be simplified if they contain **like terms**. Like terms must have the same variables and the same exponents.

Example: $3x^2$ and $-5x^2$ are like terms. $2xy$ and $7yz$ are unlike terms.

Terms are the product of a **coefficient** and a variable. In an algebraic expression, terms are separated by plus or minus signs.

Example: The expression $5x^2 + 3x - 7$ has three terms: $5x^2$, $3x$, and -7. In these terms: 5, 3, and -7 are coefficients, while x is the variable.

Algebraic expressions with one or more terms are called **polynomials.** Simple polynomials are given special names.

Monomial → one term → $8x$

Binomial → two terms → $5x - 9$

Trinomial → three terms → $6x^2 - 8x + 12$

Algebraic expressions can be multiplied by a constant. This is done by expanding using the **distributive property**.

$$a(b + c) = ab + ac$$

Example 1

Simplify $2c + 3c + 4$.

Solution

$2c + 3c + 4 = 5c + 4$ Collect like terms.

Example 2

Simplify $(2x^2 + 3) + (-4x^2 + 8)$.

Solution

$(2x^2 + 3) + (-4x^2 + 8)$

$= 2x^2 + 3 - 4x^2 + 8$ Rearrange.

$= 2x^2 - 4x^2 + 3 + 8$ Collect like terms.

$= -2x^2 + 11$

Example 3

Simplify $2(2a + b) - 3(3a - 2b)$.

Solution

$2(2a + b) - 3(3a - 2b)$ Expand.

$= 4a + 2b - 9a + 6b$ Rearrange.

$= 4a - 9a + 2b + 6b$ Collect like terms.

$= -5a + 8b$

Practise

1. Identify the variable and the coefficient in each expression.
   (a) $5x^3$ (b) $-13a$ (c) $7c^4$
   (d) $-1.35m$ (e) $\frac{4}{7}y$ (f) $\frac{5x}{8}$

2. Which terms are like terms?
   (a) $a, 5x, -3a, 12a, -9x$
   (b) $c^2, 6c, -c, 13c^2, 1.25c$
   (c) $3xy, 5x^2y, -3xy, 9x^2y, 12x^2y$
   (d) $x^2, y^2, 2xy, -y^2, -x^2, -4xy$

3. Identify each polynomial as a monomial, binomial, or trinomial
   (a) $6x^3 - 5x$ (b) $5x^3y$
   (c) $7 + 3x - 4x^2$ (d) $-yxz^3$
   (e) $5x - 2y$ (f) $3a + 5c - 4b$

4. Simplify.
   (a) $4x + 5x - 6x$ (b) $3a - 7a + 12a$
   (c) $4c + 7c - 15c$ (d) $6x^2 - 8x^2 + 3x^2$
   (e) $5xy + 7xy + 9xy$
   (f) $2a + 6b - 5a - 3b$
   (g) $3c + 8m - 10m + 5c$
   (h) $6x^2 - 3x - 8x^2 + 2x$
   (i) $4x^2 - 5x^3 + 7x^2$
   (j) $6x + 5 + 7x - 9$
   (k) $5 - 7x + 6y - 8x + 2 - 8y$
   (l) $7xy - 8x^2 + 6xy - 2x^2 - 12xy + 10x^2$
   (m) $5x - x^3 + 4x^2 - 7x^2 + 6x^3 - x$
   (n) $7x^2y - 8xy^2 + 4x^2y - 5x^2y^2$
   (o) $8x - 9y + 2z - 8z + 5x - 12y + 7$
   (p) $x^{-2} + 5x^2 - 8x^2 + 6x^{-2} + 3x^3$

5. Expand.
   (a) $2(3x - 5y + 2)$
   (b) $-3(4x + 5 - 2x^2)$
   (c) $8(3a - 5c + 6b)$
   (d) $-4(-3g + 2h - 7)$
   (e) $5(4t - v + 2r^2)$
   (f) $-5(-x - y + z)$
   (g) $-7(x^2 - 2y^2 + x^3)$
   (h) $12(3a + 2b - 6c + 2)$

6. Simplify.
   (a) $(4x - 5y) + (6x + 3) - (7x - 2y)$
   (b) $(4x + 9y) - (5x - 7y) + (2x + 5y)$
   (c) $(2a - 8ab) - (7b + 9a) + (ab - 2a + 6b)$
   (d) $(9x^2 + 2x + 2y) + (-5y - 6x^2 - 7x) - (5x^2 - 2x + 4y)$
   (e) $(7x^2 - 3x + y) - (8y - 2x^2 + 5x) - (2x^2 - 10x + 14y)$

7. Simplify.
   (a) $(2x - 5) + (8x + 13)$
   (b) $(3x + 8y) - (5x - 7y)$
   (c) $(5a - 7ab) + (6b + 4a) - (9ab - 3a + 3b)$
   (d) $3(3x - 8) - 4(8x + 1)$
   (e) $-2(4x + 5y) -4(8x - 7y)$
   (f) $5(7xy - 4x + 8y) - (6x - 9yx + 2y)$
   (g) $2(7x^2 + 3x + 5y) + 3(-2y - 9x^2 + 4x)$
   (h) $(3d^3 - 6 + 5d^2) + 4(9 - 2d^3 - 4d^2)$
   (i) $2(9a - 7ab) - 3(6b + 8a) - 4(5ab - 2a + 9b)$
   (j) $-7(x^2 + 6x + 9y) + 5(-9y -2x^2 + x)$
   (k) $6(2d^3 - 1 + 5d^2) - 5(10 - 3d^3 - 8d^2)$
   (l) $-4(9xy - 2x + 5y) -2(6x - 12yx + 12y)$

Graphing Linear Relationships

The graph of a linear relationship ($ax + by + c = 0$) is a straight line. The graph can be drawn if at least two ordered pairs of the relationship are known. This information can be determined several different ways.

Example 1: Table of Values

Sketch the graph of $2y = 4x - 2$.

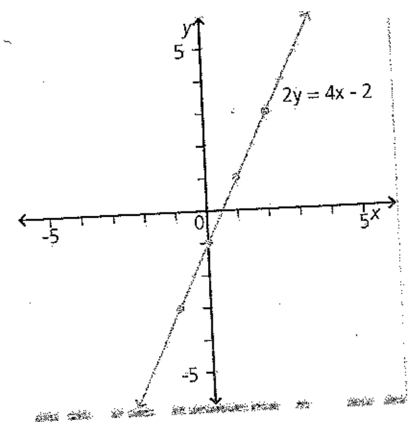
Solution

A table of values can be created.

Express the equation in the form $y = mx + b$.

$$\frac{2y}{2} = \frac{4x-2}{2}$$
$$y = 2x - 1$$

x	y
–1	2(–1) – 1 = –3
0	2(0) – 1 = –1
1	2(1) – 1 = 1
2	2(2) – 1 = 3



Example 2: Using Intercepts

Sketch the graph of $2x + 4y = 8$.

Solution

The intercepts of the line can be found.

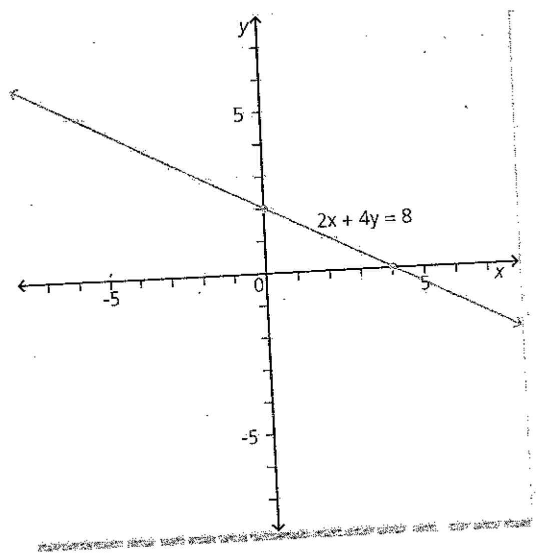
For the x-intercept, let $y = 0$.

$$2x = 8$$
$$x = 4$$

For the y-intercept, let $x = 0$.

$$4y = 8$$
$$y = 2$$

x	y
4	0
0	2



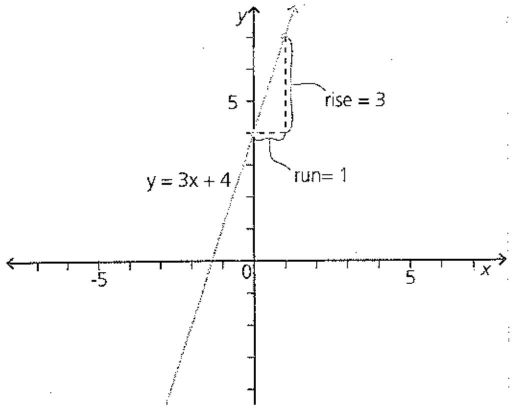
Example 3: Using the Slope and y-intercept

Sketch the graph of $y = 3x + 4$.

Solution

When the equation is in the form $y = mx + b$, the slope and y-intercept can be determined.

For $y = 3x + 4$, the line has a slope of 3 and a y-intercept of 4.



Practise

1. Express each equation in the form $y = mx + b$.
   (a) $3y = 6x + 9$
   (b) $2x - 4y = 8$
   (c) $3x + 6y - 12 = 0$
   (d) $5x = y - 9$
   (e) $2x - 5y = 20$
   (f) $4x - y - 6 = 0$
   (g) $2x + 2y = 2$
   (h) $5x - 10 = -3y$

2. Graph each equation using a table of values where $x \in \{-2, -1, 0, 1, 2\}$.
   (a) $y = 3x - 1$ (b) $y = 5x + 2$
   (c) $y = \frac{1}{2}x + 4$ (d) $y = \frac{2x + 4}{2}$
   (e) $2y = 4x + 8$ (f) $2x + 3y = 6$
   (g) $y = 4$ (h) $x = -5$

3. Determine the x- and y-intercepts of each equation.
   (a) $x + y = 10$
   (b) $2x + 4y = 16$
   (c) $5x - 7y = 35$
   (d) $9x = 54 - 6y$
   (e) $36 = 9y - 4x$
   (f) $50 - 10x - y = 0$
   (g) $\frac{x}{2} + \frac{y}{4} = 1$
   (h) $\frac{x}{5} - \frac{y}{10} = 2$

4. Graph each equation by determining the intercepts.
   (a) $x + y = 4$ (b) $x - y = 3$
   (c) $2x + y = 6$ (d) $-x + 4y = 8$
   (e) $2x + 5y = 10$ (f) $3x - 4y = 12$
   (g) $2x - 4y = -8$ (h) $-7x - 3y = 21$

5. Graph each equation using the slope and y-intercept.
   (a) $y = 2x + 3$ (b) $y = -x - 5$
   (c) $y = \frac{2}{3}x + 1$ (d) $y = -\frac{3}{4}x - 2$
   (e) $2y = x + 6$ (f) $2x + 3y = -6$
   (g) $8 - x = 4y$ (h) $x + y + 1 = 0$

6. Graph each equation. Use the most suitable method.
   (a) $y = 5x + 2$ (b) $3x - y = 6$
   (c) $y = -\frac{2}{3}x + 4$ (d) $4x = 20 - 5y$

Creating Scatter Plots and the Line of Best Fit

A **scatter plot** is a graph that shows the relationship between two sets of numeric data. The points in a scatter plot often show a general pattern or **trend**. A line that approximates a trend for the data in a scatter plot is called a **line of best fit**.

A line of best fit passes through as many points as possible, with the remaining points grouped equally above and below the line.

Data that has a **positive correlation** has a pattern that slopes up and to the right. Data that has a **negative correlation** has a pattern that slopes down and to the right. If the points nearly form a line, then the correlation is strong. If the points are dispersed, but still form some linear pattern, then the correlation is weak.

Example 1

(a) Make a scatter plot of the data and describe the kind of correlation the scatter plot shows.

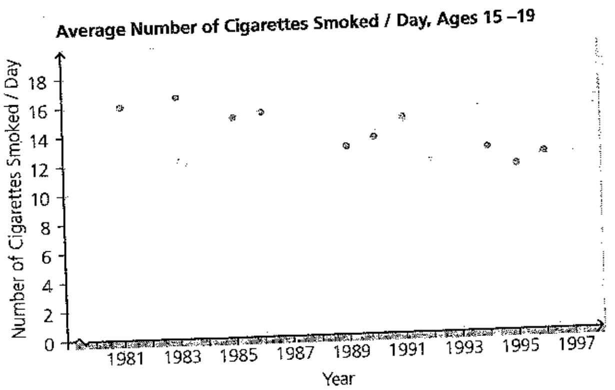
(b) Draw the line of best fit.

Long-Term Trends in Average Cigarettes Per Day by Smokers Aged 15–19

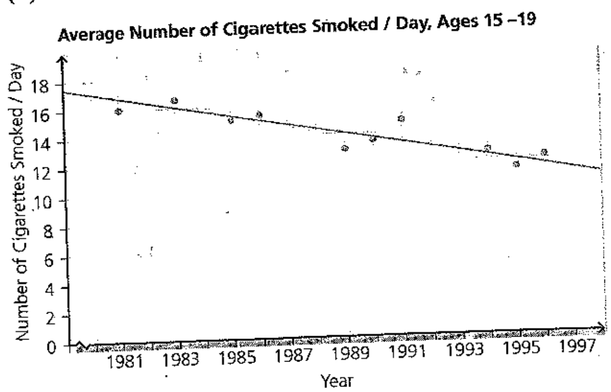
Year	1981	1983	1985	1986	1989	1990	1991	1994	1995	1996
Number Per Day	16.0	16.6	15.1	15.4	12.9	13.5	14.8	12.6	11.4	12.2

Solution

(a)



(b)



The scatter plot shows negative correlation.

Practise

1. For each set of data
   - **i.** create a scatter plot and draw the line of best fit
   - **ii.** describe the type of correlation the trend in the data displays

   (a) Property Crimes Reported in Toronto 1992–1998

Year	1992	1993	1994	1995	1996	1997	1998
Number of Crimes	5188	4839	4495	4494	4314	3932	3354

Source: Uniform Crime Reporting Survey, Canadian Centre for Justice Statistics

   (b) Percentage of Canadian Households Owning Televisions

Year	1965	1970	1975	1980	1985	1990	1997
Percentage of Households	92.6	96.0	96.8	97.7	98.3	99.0	99.1

Source: Statistics Canada

   (c) Population of the Region Hamilton-Wentworth, Ontario

Year	1966	1976	1986	1996	1998
Population	449 116	529 371	557 029	624 360	618 658

Source: Census of Canada, Statistics Canada

   (d) Percentage of Canadians with Less than Grade 9 Education

Year	1976	1981	1986	1991	1996
Percentage of the Population	25.4	20.7	17.7	14.3	12.4

Source: Census of Canada, Statistics Canada