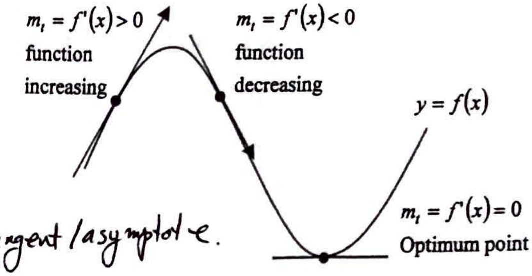
Increasing and Decreasing Functions, Local Extreme values

Date:

General rules

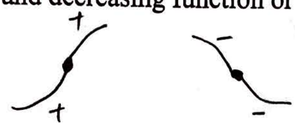


1) The function is increasing when $f'(x) > 0$.
   The function is decreasing when $f'(x) < 0$.

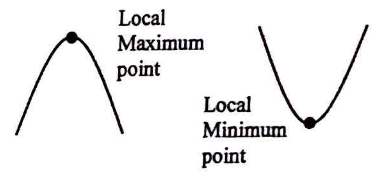
First Derivative Test:

2) The critical numbers are values of x when
   i) $f'(x) = 0$, or
   ii) $f'(x) = DNE$ (denominator of $f'(x) = 0$). ↳ cusp / v. tangent / asymptote.

3) The Local maximum/minimum points are turning points occurring between increasing and decreasing function or vice versa.



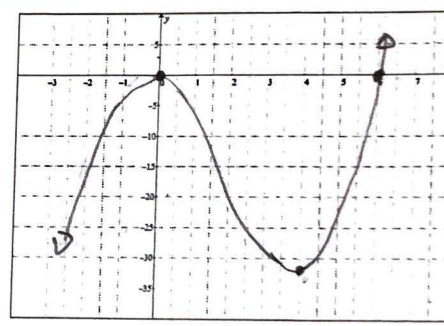
no L.Max / Min



Example 1: Increasing and decreasing function and its local extreme values

Given $f(x) = x^3 - 6x^2$. $\Rightarrow Y = x^2(x-6)$

a) Find the critical numbers.
b) Find the intervals of increase and decrease.
c) Find the local maximum and local minimum points.
d) Sketch the graph



a) $f'(x) = 3x^2 - 12x$, set $f' = 0$

$0 = 3x(x-4)$

critical #'s are: $x = 0, 4$

b)

	$x<0$	$x=0$	$0<x<4$	$x=4$	$x>4$
f'	+	0	−	0	+
graph f	↗	L.Max	↘	L.Min	↗

IOI: $x<0, x>4$

IOD: $0<x<4$

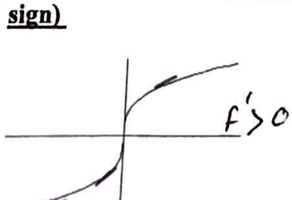
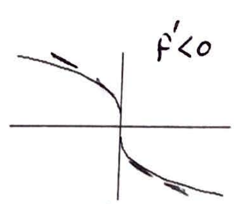
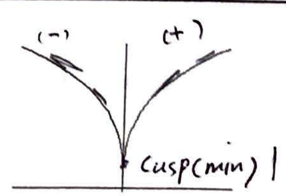
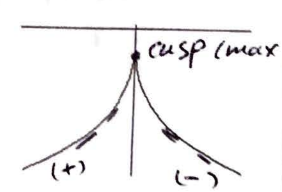
c) $f(0) = 0$, $f(4) = -32$

∴ L.Max $(0,0)$, L.Min $(4,-32)$

x-intercepts $(0,0)$, $(6,0)$

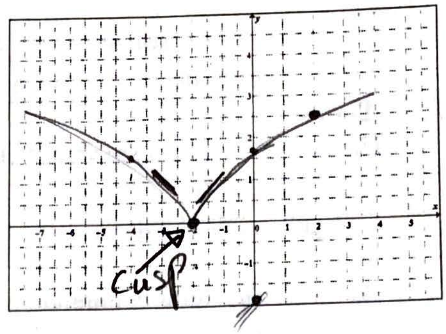
Vertical Tangents and Cusps

There are four possibilities for unbounded behavior of a derivative $f'(x)$ around a given real number **C**, all of them occur when the critical numbers x are obtained from $f'(x)$ **DNE.** The four possible cases are:

Cases 1 & 2: Vertical Tangents (limits have same sign)	**Cases 3 & 4: Cusps (limits differ in sign)**
 $\lim_{x\to C^+} f'(x) = \lim_{x\to C^-} f'(x) = +\infty$  $\lim_{x\to C^+} f'(x) = \lim_{x\to C^-} f'(x) = -\infty$	 $\lim_{x\to C^+} f'(x) = +\infty$ $\lim_{x\to C^-} f'(x) = -\infty$  $\lim_{x\to C^+} f'(x) = -\infty$ $\lim_{x\to C^-} f'(x) = +\infty$

Example 2: Vertical Tangent or Cusps

Repeat example 1 for $f(x)=(x+2)^{\frac{2}{3}}$



a) $f'(x) = \frac{2}{3}(x+2)^{-\frac{1}{3}}$, set $f'=0$

$0 = \frac{2}{3(x+2)^{1/3}}$

$f' \neq 0$, $f' =$ DNE when $x = -2$

	$x<-2$	$x=-2$	$x>-2$
f'	$-$	DNE	$+$
f	↘	cusp	↗

IOI: $x>-2$
IOD: $x<-2$
L. Min (cusp) @ $(-2,0)$

y-int: $(0, 1.6)$

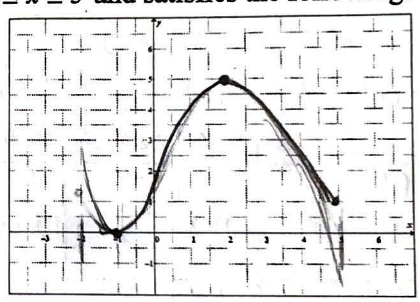
$f(-4) = 1.6$

$(-4+2)^{(2/3)} = \left[(-2)^{1/3}\right]^2$

Example 3: Sketching a graph with given information

Sketch a graph of a function that is differentiable on the interval $-2 \le x \le 5$ and satisfies the following.

i) graph of f passes through points (-1,0) and (2,5);
ii) function f decreasing on $-2 < x < -1$, increasing on $-1 < x < 2$, decreasing again on $2 < x < 5$.



Example 4: Determine the function

Find values for a, b, and c such that the graph of $f(x) = ax^2 + bx + c$ has a relative maximum at (3, 12) and crosses the y-axis at (0, 1).

Given: $f'(3)=0 \to f'(x) = 2ax+b \to 0 = 2a(3)+b$
$0 = 6a+b$
$b = -6a$ ①

$f(0)=1 \to 1 = 0+0+c$
$c=1$

$f(3)=12$
$\to 12 = a(9)+b(3)+1$
$11 = 9a+3b$ ②

sub ① → ②
$11 = 9a + 3(-6a)$
$11 = 9a - 18a$
$11 = -9a$
$a = \frac{-11}{9}$

→ ① $b = -\cancel{6}^{2}\left(\frac{-11}{\cancel{9}_{3}}\right)$

$b = \frac{22}{3}$

Homework:
P. 169 #1,3,5,7,8
P. 178 #5,7,8,9,11,15