

Solve for x in the interval $[-\pi, 0]$

(a) $\cos x = -\frac{1}{\sqrt{2}}$

$\cos x < 0$

$x \in QII$ or $x \in QIII$

RAA = $\cos^{-1}(\frac{1}{\sqrt{2}}) = \frac{\pi}{4}$

In QII : $x = \pi - \frac{\pi}{4}$

$x = \frac{3\pi}{4}$

In $QIII$: $x = \pi + \frac{\pi}{4}$

$x = \frac{5\pi}{4}$

period = 2π

$x_1 = \frac{3\pi}{4} - 2\pi = -\frac{5\pi}{4}$ (outside of bounds) \times

$x_2 = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$ \checkmark

Answer: $x = -\frac{3\pi}{4}$

(b) $\tan^2 x = \tan x$

Do not divide both sides by $\tan x$, solutions will be lost if you do!

$\tan^2 x - \tan x = 0$

$\tan x (\tan x - 1) = 0$

$\tan x = 0$ or $\tan x - 1 = 0$

$x = 0$ or $\tan x = 1 > 0$

or

$x = \pi$

RAA = $\tan^{-1}(1) = \frac{\pi}{4}$

$x \in QI$ or $x \in QIII$

so $x = \frac{\pi}{4}$ or $x = \pi + \frac{\pi}{4}$

$x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

Subtract multiple of π , where π is the period!

$x_1 = 0 - \pi = -\pi$

$x_2 = \pi - \pi = 0$

$x_3 = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$

$x_4 = \frac{5\pi}{4} - 2\pi = -\frac{3\pi}{4}$

Answers: $0, -\pi, -\frac{3\pi}{4}$

$\sin^2 x - \sin x = 2$

$\sin^2 x - \sin x - 2 = 0$

let $\sin x = a, -1 \leq a \leq 1$

$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a-2=0$ or $a+1=0$

$a=2$ or $a=-1$

↑ not admissible, since $-1 \leq a \leq 1$

$\sin x = -1$

$x = \frac{3\pi}{2}$

period

Then $x = \frac{3\pi}{2} - 2\pi = -\frac{\pi}{2}$

Answer: $x = -\frac{\pi}{2}$

$\sin^2 x = \frac{3}{4}$

$\sin x = \pm \sqrt{\frac{3}{4}}, \sin x = \pm \frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$ or $\sin x = -\frac{\sqrt{3}}{2}$

$\sin x > 0$

RAA = $\sin^{-1}(\frac{\sqrt{3}}{2})$

RAA = $\frac{\pi}{3}$

QI, QII

$x = \frac{\pi}{3}, x = \frac{2\pi}{3}$

period = 2π

$x = \frac{\pi}{3} - 2\pi = -\frac{5\pi}{3}$ \times

$x = \frac{2\pi}{3} - 2\pi = -\frac{4\pi}{3}$ \times

$\sin x < 0$

RAA = $\sin^{-1}(\frac{\sqrt{3}}{2})$

RAA = $\frac{\pi}{3}$

$QIII, QIV$

$x = \frac{4\pi}{3}, x = \frac{5\pi}{3}$

$x = \frac{4\pi}{3} - 2\pi = -\frac{2\pi}{3}$ \checkmark

$x = \frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$ \checkmark

Answer: $-\frac{\pi}{3}, -\frac{2\pi}{3}$