

④

If $\cos \theta + \sin \theta = \frac{2}{3}$, find $\sin 2\theta$.

$$[(\cos \theta) + (\sin \theta)]^2 = \frac{4}{9}$$

$$\cos^2 \theta + 2\cos \theta \sin \theta + \sin^2 \theta = \frac{4}{9}$$

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$1 + \sin 2\theta = \frac{4}{9}$$

$$\sin 2\theta = \frac{4}{9} - 1$$

$$\sin 2\theta = -\frac{5}{9} \checkmark$$

⑤

If $2a + b = \frac{\pi}{2}$, prove $\cos a = \pm \sqrt{\frac{1 + \sin b}{2}}$

given.

$$\Rightarrow b = \frac{\pi}{2} - 2a,$$

$$\sin b = \sin\left(\frac{\pi}{2} - 2a\right) = \cos 2a.$$

$$\sin b = \cos 2a = 2\cos^2 a - 1.$$

$$\sin b = 2\cos^2 a - 1$$

$$1 + \sin b = 2\cos^2 a$$

$$\cos^2 a = \frac{1 + \sin b}{2}$$

$$\cos a = \pm \sqrt{\frac{1 + \sin b}{2}}$$