Textbook: p.29 #14

$f(x)=x^3+2x^2-5x-6$

(a) degree = 3, odd degree: opposite end behaviour

$a=1>0$, from QIII into QI

$\left.\begin{array}{l}\text{As } x\to+\infty,\ y\to+\infty \\ \text{As } x\to-\infty,\ y\to-\infty\end{array}\right]$ ← Actual end behaviour.

(b) Need zeroes:

Let $f(x)=0$, $0=x^3+2x^2-5x-6$.

Let's hope there are integer solutions. If it is the case:

$$0=x^3+2x^2-5x-6$$

$$6=x^3+2x^2-5x \quad \text{and we factor } x \text{ on Right Side}$$

$$6=x(x^2+2x-5)$$

Then x is a factor of 6. Let's list integer factors of 6:

$\pm1, \pm2, \pm3, \pm6$

Now we check those values:

$f(-1)=0$ so $(x-(-1))=(x+1)$ is a linear factor of $f(x)$

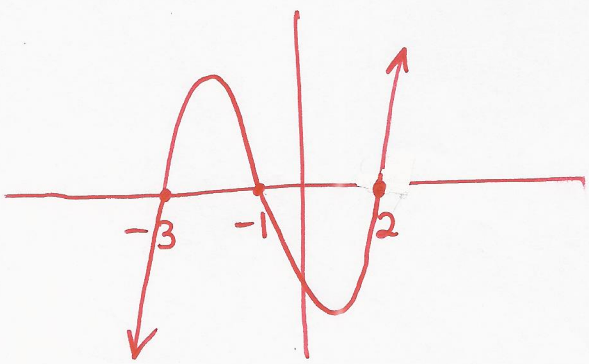
To every x-intercept, $x=a$, there corresponds a linear factor $(x-a)$ of $f(x)$.

$f(2)=0$ so $(x-2)$ is a factor

$f(-3)=0$ so $(x+3)$ is a factor

$f(x)=a(x-b)(x-c)(x-d)$

$f(x)=(x+1)^1(x-2)^1(x+3)^1$ ← each zero is of order 1, so the graph crosses (and looks linear "for a moment") at $x=-3$, $x=-1$, $x=2$



Good idea:

Check on each interval

(a) pick $x<-3$, say $x=-4$, $y<0$

(b) pick $-3<x<-1$, say $x=-2$, $y>0$

(c) pick $-1<x<2$, say $x=0$, $y<0$

(d) pick $x>2$, say $x=3$, $y>0$