p.112 #24

Determine the product of all values of k for which the polynomial equation

$2x^3-9x^2+12x-k=0$ has a double root.

Solution:

Let $x=a$ represent the double root

Then $(x-a)^2$ is a factor of the polynomial.

We need the leading coefficient of 2.

Let $(2x-b)$ represent the remaining third factor.

Then

$(x-a)^2(2x-b)=2x^3-9x^2+12x-k$

We can use the method of compairing (equating) coefficients (with two different representations).

$(x^2-2ax+a^2)(2x-b)=2x^3-9x^2+12x-k$

$2x^3-bx^2-4ax^2+2abx+2a^2x-a^2b=2x^3-9x^2+12x-k$

$2x^3-(b+4a)x^2+(2ab+2a^2)x-a^2b=2x^3-9x^2+12x-k$

$\begin{cases} b+4a=9 \\ 2ab+2a^2=12 \\ k=a^2b \end{cases} \rightarrow \begin{cases} 4a+b=9 \quad (1) \rightarrow b=9-4a \\ a^2+ab=6 \quad (2) \\ k=a^2b \quad (3) \end{cases}$

Sub into (2):

$a^2+a(9-4a)=6$

$a^2-4a^2+9a=6$, $-3a^2+9a-6=0$, $a^2-3a+2=0$

$(a-2)(a-1)=0 \Rightarrow a-2=0$ or $a-1=0 \Rightarrow a=2$ or $a=1$

If $a=2$, $b=9-4(2)=1$; If $a=1$, $b=9-4(1)=5$

We have two cases:

Case 1: $a=2, b=1, k=(2)^2(1)=4$

Case 2: $a=1, b=5, k=(1)^2(5)=5$

The product of possible k-values is $5\cdot4=20$ ✓