

p.103 #22

When a polynomial is divided by  $(x+2)$ , the remainder is  $-19$ . When the same polynomial is divided by  $(x-1)$ , the remainder is  $2$ . Determine the remainder when the polynomial is divided by  $(x-1)(x+2)$

Let  $P(x)$  represent the polynomial (as a function)

$$\begin{cases} P(-2) = -19 \\ P(1) = 2 \end{cases}$$

The division statement gives

$$P(x) = \underbrace{(x-1)(x+2)}_{\text{divisor}} \cdot Q(x) + \underbrace{R(x)}_{\text{degree must be (at most) 1}}$$

Then  $R(x) = ax + b$

$$\begin{aligned} P(-2) &= a(-2) + b = -19; & P(1) &= a(1) + b = 2 \\ -2a + b &= -19 \quad \textcircled{1} & a + b &= 2 \quad \textcircled{2} \end{aligned}$$

This gives a linear system

$$\begin{cases} -2a + b = -19 \quad \textcircled{1} \\ a + b = 2 \quad \textcircled{2} \end{cases}$$

$$\textcircled{2} - \textcircled{1}: 3a = 21, a = 7$$

$$\text{Sub into } \textcircled{2}: 7 + b = 2, b = -5$$

Therefore the remainder is  $R(x) = 7x - 5$