

p. 93 #20

When the polynomial $mx^3 - 3x^2 + nx + 2$ is divided by $x+3$, the remainder is -1 . When it is divided by $x-2$, the remainder is -4 . Determine the values of m and n .

Solution:

$$\text{Let } P(x) = mx^3 - 3x^2 + nx + 2$$

$$\begin{cases} P(-3) = -1 \\ P(2) = -4 \end{cases}$$

$$P(-3) = m(-3)^3 - 3(-3)^2 + n(-3) + 2 = -1$$

$$-27m - 27 - 3n + 2 = -1$$

$$-27m - 3n = 24 \rightarrow \boxed{9m + n = -8} \quad (1)$$

$$P(2) = m(2)^3 - 3(2)^2 + 2n + 2 = -4$$

$$8m - 12 + 2n + 2 = -4$$

$$8m + 2n = 6 \rightarrow \boxed{4m + n = 3} \quad (2)$$

We solve the linear system

$$\begin{cases} 9m + n = -8 \quad (1) \\ 4m + n = 3 \quad (2) \end{cases}$$

$$(1) - (2): 5m = -11, \quad m = -\frac{11}{5}$$

$$\text{Sub } m = -\frac{11}{5} \text{ into } (2): 4\left(-\frac{11}{5}\right) + n = 3$$

$$n = 3 + \frac{44}{5}$$

$$n = \frac{15}{5} + \frac{44}{5} \rightarrow n = \frac{59}{5}$$

$$n = \frac{15}{5} + \frac{44}{5}$$

$$\text{Answer: } m = -\frac{11}{5}, n = \frac{59}{5}$$