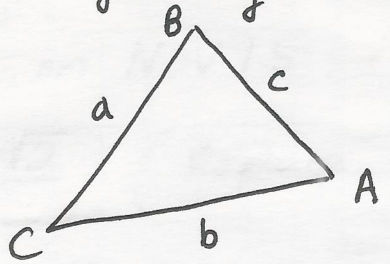
p.227

(#27) Given ΔABC with sides a, b, and c, show that the area of the triangle is given by

$$A = \frac{a^2 \sin B \sin C}{2\sin(B+C)}$$



$A = \frac{1}{2} ab\sin C$ and $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \rightarrow \boxed{b = \frac{a\sin B}{\sin A}}$

Therefore, $A = \frac{1}{2}a\left(\frac{a\sin B}{\sin A}\right)\sin C$

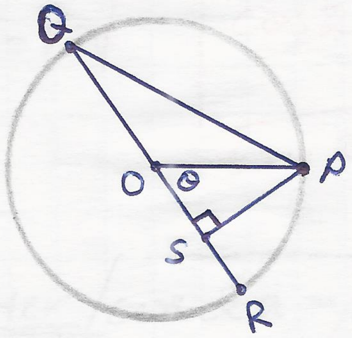
$A = \frac{1a^2 \sin B \sin C}{2\sin A}$. But $A = 180° - (B+C)$

so $\sin A = \sin[180° - (B+C)] = \sin(B+C)$

Therefore, $A = \frac{a^2 \sin B \sin C}{2\sin(B+C)}$

(#29) Use the unit circle shown to prove that

$$\tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$$



Which angle is $\frac{\theta}{2}$?

We need to find other angles in the Δs in the diagram

Since ΔQOP is isosceles, let $\angle PQO = \angle OPQ = x$

then in ΔQPS: $x + 90° + 90° - \theta + x = 180°$

$180° + 2x - \theta = 180°$, $\theta = 2x$, $x = \frac{\theta}{2}$

$\therefore \angle OQP = \angle OPQ = \frac{\theta}{2}$

In ΔQPS:

$$\tan\frac{\theta}{2} = \frac{PS}{QS} = \frac{(1)\sin\theta}{QS} = \frac{\sin\theta}{1+OS} = \frac{\sin\theta}{1+\cos\theta}.$$