

7. Prove the identity and generalize

$$\cos \frac{\theta}{2} \cos \frac{\theta}{4} \cos \frac{\theta}{8} = \frac{\sin \theta}{8 \sin \frac{\theta}{8}}$$

$$\text{L.S.} = \cos \frac{\theta}{2} \cos \frac{\theta}{4} \cos \frac{\theta}{8}$$

$$= \frac{\cos \frac{\theta}{2} \cos \frac{\theta}{4} \cos \frac{\theta}{8}}{1} \cdot \frac{2 \sin \frac{\theta}{8}}{2 \sin \frac{\theta}{8}} \quad \text{"dressing it up!"}$$

$$= \frac{\cos \frac{\theta}{2} \cos \frac{\theta}{4} \sin \frac{\theta}{4}}{2 \sin \frac{\theta}{8}} \cdot \frac{2}{2} = \frac{\cos \frac{\theta}{2} \sin \frac{\theta}{2}}{4 \sin \frac{\theta}{8}}$$

$$= \frac{2 \cos \frac{\theta}{2} \sin \frac{\theta}{2}}{8 \sin \frac{\theta}{8}} = \frac{\sin \theta}{8 \sin \frac{\theta}{8}} = \text{R.S. qed!)$$