

35. Prove  $\sec x - \tan x = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$

$$\text{R.S.} = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{x}{2}\right)}$$

$$= \frac{1 - \tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{x}{2}\right)}$$

$$= \frac{1 - \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}{1 + \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}} = \frac{\frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}{\frac{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}}$$

$$= \frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} \cdot \frac{\cos\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

$$= \frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)} = \frac{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)} \cdot \frac{[\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)]}{[\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)]}$$

$$= \frac{\cos^2\left(\frac{x}{2}\right) - 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) + \sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)} = \frac{1 - 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}{\cos\left[2\left(\frac{x}{2}\right)\right]}$$

$$= \frac{1 - \sin x}{\cos x} = \frac{1}{\cos x} - \frac{\sin x}{\cos x} = \sec x - \tan x = \text{L.S.}$$

Q.E.D.

$$\tan(A-B)$$

$$= \frac{\tan A - \tan B}{1 + \tan A \tan B}$$