

Laws of Logarithms

6 a)

Given: $\log_3 15 = a$ and $\log_3 10 = b$

$$\log_{\sqrt{3}} 50 = ?$$

Solution:

We first work with what we are given and simplify it

$$a = \log_3 15 = \log_3 (3 \cdot 5) = \log_3 3 + \log_3 5$$

$$a = 1 + \log_3 5 \Rightarrow \boxed{\log_3 5 = a - 1}$$

$$b = \log_3 10 = \log_3 (5 \cdot 2) = \log_3 5 + \log_3 2$$

we know this from before

$$b = a - 1 + \log_3 2 \Rightarrow \boxed{\log_3 2 = b - a + 1}$$

Finally,

$$\log_{\sqrt{3}} 50 = \frac{1}{\log_{50} \sqrt{3}} = \frac{1}{\log_{50} 3^{\frac{1}{2}}} = \frac{1}{\frac{1}{2} \log_{50} 3} = \frac{1}{\frac{1}{2}} \cdot \frac{1}{\log_{50} 3}$$

$$= 2 \cdot \frac{1}{\log_{50} 3} = 2 \log_3 50 = 2 (\log_3 (25 \cdot 2)) =$$

$$= 2 [\log_3 25 + \log_3 2] = 2 [\log_3 5^2 + \log_3 2]$$

$$= 2 [2 \log_3 5 + \log_3 2]$$

$$= 2 [2(a-1) + b-a+1]$$

$$= 2 [2a - 2 + b - a + 1]$$

$$= 2 [a + b - 1] \checkmark$$