(2.) $\cos^4 x - \sin^4 x = 1 - 2\sin^2 x$

$$
\begin{aligned}
L.S. &= \cos^4 x - \sin^4 x \\
&= (\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) \\
&= (\cos^2 x - \sin^2 x)(1) \\
&= \cos^2 x - \sin^2 x \\
&= 1 - \sin^2 x - \sin^2 x \\
&= 1 - 2\sin^2 x = R.S. \quad QED.
\end{aligned}
$$

(4.) $\cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x = 1$

$$
\begin{aligned}
L.S. &= \cos^2 x \cos^2 y + \sin^2 x \sin^2 y + \sin^2 x \cos^2 y + \sin^2 y \cos^2 x \\
&= \cos^2 x[\cos^2 y + \sin^2 y] + \sin^2 x[\sin^2 y + \cos^2 y] \\
&= (1)\cos^2 x + (1)\sin^2 x \\
&= \cos^2 x + \sin^2 x \\
&= 1 = R.S. \quad QED
\end{aligned}
$$

(8.) $\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$

$$
\begin{aligned}
L.S. &= \cos^6 x + \sin^6 x = (\cos^2 x)^3 + (\sin^2 x)^3 \\
&= \underbrace{(\cos^2 x + \sin^2 x)}_{1}(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x) \\
&= (1)(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x) \leftarrow \text{2nd factor is an incomplete PST.} \\
&= \cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x - 3\cos^2 x \sin^2 x \\
&= (\cos^2 x + \sin^2 x)^2 - 3\cos^2 x \sin^2 x \\
&= 1 - 3\cos^2 x \sin^2 x = 1 - 3(1 - \sin^2 x)\sin^2 x = 1 - 3\sin^2 x + 3\sin^4 x
\end{aligned}
$$