⑧ let $f(x) = x^3 + kx^2 + 2x - 3$

Since the remainder is 1 when the divisor is $(x+2)$,

$f(-2) = 1$ by the Remainder Theorem.

$(-2)^3 + k(-2)^2 + 2(-2) - 3 = 1$

$-8 + 4k - 4 - 3 = 1$

$4k = 16$, $\boxed{k = 4}$

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⑨ let $f(x) = mx^3 + g(x)^2 - x + 3$

$f(-1) = 3$, $m(-1)^3 + g(-1)^2 - (-1) + 3 = 3$

$-m + g + 4 = 3$, $\boxed{-m + g = -1}$ ①

$f(-2) = -7$

$m(-2)^3 + g(-2)^2 - (-2) + 3 = -7$

$-8m + 4g + 2 + 3 = -7$, $-8m + 4g = -12$, $\boxed{2m - g = 3}$ ②

Solving the linear system:

$\begin{cases} -m + g = -1 \\ 2m - g = 3 \end{cases}$ ← Adding the equations

$m = 2$ Sub into ①:

Then $-2 + g = -1$, $\boxed{g = 1}$

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⑩ let $P(x) = ax^3 + bx^2 - 37x + 14$

Divisor needs to be factored: $(x^2 + x - 2) = (x+2)(x-1)$

$P(x) = d(x) \cdot q(x) + r(x)$

$P(x) = (x+2)(x-1)Q(x) + 0$ ← remainder

Then $\begin{cases} P(-2) = 0 \rightarrow P(-2) = -8a + 4b + 74 + 14 = 0 \rightarrow -8a + 4b = -88 \\ P(1) = 0 \rightarrow P(1) = a + b - 37 + 14 = 0 \rightarrow a + b = 23 \end{cases}$

$\begin{cases} -2a + b = -22 & ① \\ a + b = 23 & ② \end{cases}$ $\rightarrow b = 8$

Subtracting ① from ②:

$3a = 45$

$a = 15$

From ②: $b = 23 - a$

$b = 23 - 15$

Answer:

$a = 15$

$b = 8$