

$$\text{Prove that } \sin d + \sin \theta = 2 \sin\left(\frac{d+\theta}{2}\right) \cdot \cos\left(\frac{d-\theta}{2}\right)$$

Our goal is to see if we can replace d and θ on the left side w some sum/difference and then we could use compound angle identity (which is one of our tools)

We can consider arguments/inputs of sine/cosine functions on the right side! the benefit is introducing those arguments into consideration and expressing easily d and θ .

$$\frac{d+\theta}{2} + \frac{d-\theta}{2} = d \quad (\text{simplifies to alpha!})$$

$$\frac{d+\theta}{2} - \frac{d-\theta}{2} = \theta$$

Then...

$$\text{L.S.} = \sin d + \sin \theta =$$

$$= \sin\left[\frac{d+\theta}{2} + \frac{d-\theta}{2}\right] + \sin\left[\frac{d+\theta}{2} - \frac{d-\theta}{2}\right] \quad \text{and} \quad \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$= \sin\left(\frac{d+\theta}{2}\right) \cos\left(\frac{d-\theta}{2}\right) + \cancel{\cos\left(\frac{d+\theta}{2}\right) \sin\left(\frac{d-\theta}{2}\right)} + \sin\left(\frac{d+\theta}{2}\right) \cos\left(\frac{d-\theta}{2}\right) -$$

$$\cancel{\cos\left(\frac{d+\theta}{2}\right) \sin\left(\frac{d-\theta}{2}\right)} = 2 \sin\left(\frac{d+\theta}{2}\right) \cos\left(\frac{d-\theta}{2}\right)$$