How To Graph a 'Simple' Rational Function

1) If the function is in the form $h(x)=\frac{a}{b(x-h)}+k$ or $h(x)=\frac{a}{[b(x-h)^2]}+k$, then graph by applying appropriate transformations to either the graph of $y=\frac{1}{x}$ or $y=\frac{1}{x^2}$.

2) If the function is in the form $h(x)=\frac{1}{g(x)}$, then graph y = g(x) and systematically reciprocate the y-coordinates and plot. Recall that the zeros of y = g(x) become V.A.s for the graph of y = h(x) and these are good features to determine first. (Note: if the function is of the form $h(x)=\frac{a}{g(x)}$, then graph $h(x)=\frac{1}{g(x)}$ and perform a stretch/reflection.)

3) If the function cannot be graphed using the above techniques, follow the following steps.

Step 0: **Simplify by factoring and reducing, if possible. State restrictions.** Simplifying the rational function may put it in form stated above. Graph accordingly keeping in mind any point discontinuities.

Step 1: **Determine the x-intercept(s) and plot.**
(After simplifying the function, set the numerator to 0 and solve)

Step 2: **Determine the y-intercept and plot.**
(Let x = 0 and evaluate)

Step 3: **Determine the equations of any Vertical Asymptotes.**
Graph V.A.s using dotted lines. *(Set the denominator to 0 and solve)*

Step 4: **Determine whether the function shoots up to + ∞ or down to − ∞ near its vertical asymptotes.** (Consider the signs of each factor of the function for x values close to both sides of each V.A.)

Step 5: **Determine the equation of any Horizontal Asymptote or Oblique Asymptote.**
Graph any H.A. or O.A. using a dotted line. *(Examine the degrees of the numerator and denominator and use a shortcut to find the equation of any H.A.. Use long division for O.A.)*

Step 6: **Determine if there are any holes (point discontinuities) for your graph.** Plot as a tiny circle at the location of the hole. *(A hole occurs at restricted value of x that came from a factor in the original rational function that was divided out when simplifying. To find the location of a hole, substitute the restricted value of x into the simplified version of the function to see where the point would have been.)*

Step 7: **If necessary, find other points on the graph** to determine the general shape and/or the behaviour of the graph near its horizontal asymptotes. *(Sub in values for x and find y)*