

Determine $\log_{49} 16$ if $\log_{14} 28 = a$

Solution: We start with given

$\log_{14} 28 = a$ and simplify, noting

that both 14 and 49 are multiples of 7

$$\log_{14} 28 = \frac{\log_7 28}{\log_7 14} = \frac{\log_7 7 + \log_7 4}{\log_7 7 + \log_7 2}$$

$$\therefore \log_{14} 28 = \frac{1 + \log_7 2^2}{1 + \log_7 2} = \frac{1 + 2\log_7 2}{1 + \log_7 2} = a$$

We can express $\log_7 2$ in terms of a now: $1 + 2\log_7 2 = a + a\log_7 2$

$$2\log_7 2 - a\log_7 2 = a - 1$$

$$(\log_7 2)(2 - a) = a - 1 \rightarrow \boxed{\log_7 2 = \frac{a - 1}{2 - a}}$$

Now:

$$\begin{aligned} \log_{49} 16 &= \log_{49} 2^4 = 4 \log_{49} 2 = 4 \cdot \frac{1}{\log_2 49} \\ &= 4 \cdot \frac{1}{2 \log_2 7} = (2) \cdot \frac{1}{\log_2 7} = 2 \log_7 2 \end{aligned}$$

$$\Rightarrow \log_{49} 16 = \frac{2(a - 1)}{2 - a}$$