

Prove

$$\cos(a+b)\cos(a-b) = \cos^2 a - \sin^2 b$$

$$\text{L.S.} = \cos(a+b)\cos(a-b)$$

$$= (\cos a \cos b - \sin a \sin b)(\cos a \cos b + \sin a \sin b)$$

$$= \cos^2 a \cos^2 b + \cos a \cos b \sin a \sin b$$

$$- \cos a \cos b \sin a \sin b - \sin^2 a \sin^2 b$$

$$= \cos^2 a \cos^2 b - \sin^2 a \sin^2 b$$

$$= \cos^2 a (1 - \sin^2 b) - (1 - \cos^2 a) \sin^2 b$$

$$= \cos^2 a - \cos^2 a \sin^2 b - \sin^2 b + \cos^2 a \sin^2 b$$

$$= \cos^2 a - \sin^2 b$$