

Example:

Express $\sin \theta$, $\cos \theta$ and $\tan \theta$ in terms of $\tan\left(\frac{\theta}{2}\right)$.

(What it means is to obtain equations

$\left. \begin{array}{l} \sin \theta = \dots \\ \cos \theta = \dots \\ \tan \theta = \dots \end{array} \right\}$ where \dots is a stand in for expressions involving $\tan\left(\frac{\theta}{2}\right)$ and numbers only.)

$$\begin{aligned} \textcircled{1} \sin \theta &= \sin\left[2\left(\frac{\theta}{2}\right)\right] = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}} \\ &= \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}} = \left\{ \begin{array}{l} \text{divide num-r} \\ \text{and denom-r} \\ \text{by } \cos^2\left(\frac{\theta}{2}\right) \end{array} \right\} = \frac{2\tan\left(\frac{\theta}{2}\right)}{1 + \tan^2\frac{\theta}{2}} \quad \checkmark \end{aligned}$$

↑ same angle ↑

$$\textcircled{2} \text{ Similarly } \cos \theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}$$

$$\textcircled{3} \tan \theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}}$$