① Prove that

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

② Use the formula above to show that the least value of $\sin\theta + \cos\theta$ is $-\sqrt{2}$ and the greatest value is $\sqrt{2}$.

$\boxed{\text{Solution:}}$

① let $\frac{A+B}{2} = x$, $\frac{A-B}{2} = y$. Then $\begin{cases} x+y = A \\ x-y = B \end{cases}$

and

$$\sin A + \sin B = \sin(x+y) + \sin(x-y)$$

$$= \sin x\cos y + \cancel{\cos x\sin y} + \sin x\cos y - \cancel{\cos x\sin y}$$

$$= 2\sin x\cos y = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$$

② We transform the given sum into a product:

$$\sin\theta + \cos\theta = \sin\theta + \sin\left(\frac{\pi}{2} - \theta\right)$$

$$= 2\sin\frac{\pi}{4}\cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}\cos\left(\theta - \frac{\pi}{4}\right)$$

Since $-1 \le \cos\square \le 1$, $-\sqrt{2} \le (\cos\square)\sqrt{2} \le \sqrt{2}$

$$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$$