

If $\tan x = \frac{4}{3}$, $\pi < x < \frac{3\pi}{2}$, find

the value of $\tan\left(\frac{x}{2}\right)$.

$$\tan x = \tan\left[2\left(\frac{x}{2}\right)\right] = \frac{2 \tan \frac{x}{2}}{1 - \tan^2\left(\frac{x}{2}\right)} = \frac{4}{3}$$

$$6 \tan \frac{x}{2} = 4 - 4 \tan^2\left(\frac{x}{2}\right)$$

$$4 \tan^2\left(\frac{x}{2}\right) + 6 \tan\left(\frac{x}{2}\right) - 4 = 0$$

$$2\left(2 \tan^2\left(\frac{x}{2}\right) + 3 \tan\left(\frac{x}{2}\right) - 2\right) = 0$$

$$2 \tan^2\left(\frac{x}{2}\right) + 3 \tan\left(\frac{x}{2}\right) - 2 = 0$$

$$(2 \tan\left(\frac{x}{2}\right) - 1)(\tan\left(\frac{x}{2}\right) + 2) = 0$$

$$2 \tan\left(\frac{x}{2}\right) - 1 = 0 \text{ or } \tan\left(\frac{x}{2}\right) + 2 = 0$$

$$\tan\left(\frac{x}{2}\right) = \frac{1}{2} \text{ or } \tan\left(\frac{x}{2}\right) = -2$$

$$\pi < x < \frac{3\pi}{2} \rightarrow \frac{\pi}{2} < \frac{x}{2} < \frac{3\pi}{4}$$

Q II

$$\tan\left(\frac{x}{2}\right) < 0 \rightarrow \tan\left(\frac{x}{2}\right) = -2 \checkmark$$