

MHF p166 #11 (HW) (Section 3.2)

Each function described below is the reciprocal of a quadratic function. Write an equation to represent each function.

(a) The horizontal asymptote is $y=0$. The vertical asymptotes are $x=2$ and $x=-3$. The intervals $x < -3$ and $x > 2$ have $y > 0$.

Example: $y = \frac{1}{(x-2)(x+3)}$

The horizontal asymptote, which is about the behaviour in the long run, is $y=0$. That means as $x \rightarrow \pm\infty$, the denominator "approaches" $+\infty$, so $f(x) \rightarrow 0^+$

[Indeed as $\underbrace{x \rightarrow -\infty}_{x < -3}$, $y > 0$; as $\underbrace{x \rightarrow +\infty}_{x > 2}$, $y > 0$]

Answers may vary and variance has to do with a constant multiplier; e.g.

$$y = \frac{5}{(x-2)(x+3)}$$

(b) The horizontal asymptote is $y=0$. There is no vertical asymptote. The max point is $(0, 0.5)$
Domain: $\{x | x \in \mathbb{R}\}$

$$y = \frac{1}{f(x)} \leftarrow \text{reciprocal function with H.A. } y=0$$

No vertical asymptote means the denominator, $f(x)$ has no real roots. So $f(x)$ has no real x -intercepts.

For the reciprocal to have a maximum, the original function, $f(x)$, has to have a min. value of $\frac{1}{0.5} = 2$.

$\therefore y = \frac{1}{x^2 + 2}$. The max point $(0, 0.5)$ corresponds to the vertex of "original" function $f(x)$ which is $(0, 2)$

(c) The horizontal asymptote is $y=0$

The vertical asymptote is $x=-3$.

Domain: $\{x \in \mathbb{R}, x \neq -3\}$

Since HA: $y=0$ is present, as $x \rightarrow \pm\infty$,
 $y \rightarrow 0$.

Using information about asymptotes

$$y = \frac{1}{(x+3)^2} \quad \text{or} \quad y = -\frac{1}{(x+3)^2}$$

or multiples of those.