

Exponential Equations

Simplest exponential equations are solved using the following property:

If $a^{x_1} = a^{x_2}$ then $x_1 = x_2$

From equality of single powers with the same base (and coefficient of 1), equality of exponents follows!

$|a^x = |a^y$ means $x = y$ ← we sometimes refer to this result briefly by writing BATS

Examples:

(a) Solve $4^x = 8^{x+3}$ Aim to have same bases.

Both 4 and 8 are powers of 2 so

$$(2^2)^x = (2^3)^{x+3} \leftarrow \text{note brackets implied!}$$

$$2^{2x} = 2^{3x+9}, \text{ BATS: } 2x = 3x + 9 \\ -x = 9, \boxed{x = -9}$$

$$(b) 300(2)^{x+1} = 600$$

Remark: cannot multiply 300 and 2, 300 is a number (coefficient) while 2 is a base.

Divide both sides by 300:

$$2^{x+1} = 2^{\leftarrow \text{implied}} \text{, BATS! } x+1 = 1, \boxed{x = 0}$$

$$(c) 2^{x+2} - 2^x = 12$$

Note 2^{x+2} is divisible by 2^x or 2^x is a factor of 2^{x+2}

$$2^x \cdot 2^2 - 2^x = 12$$

$$2^x(2^2 - 1) = 12, \quad 2^x(4 - 1) = 12$$

$$2^x(3) = 12, \quad 2^x = 4, \quad 2^x = 2^2, \text{ BATS! } x = 2$$