$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$	$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Example 1: Prove that $\cos\frac{\pi}{5}\cos\frac{2\pi}{5} = \frac{1}{4}$

$$L.S. = \cos\frac{\pi}{5}\cos\frac{2\pi}{5} = \frac{2\sin\frac{\pi}{5}\cos\frac{\pi}{5}\cos\frac{2\pi}{5}}{2\sin\frac{\pi}{5}} = \frac{\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}}{2\sin\frac{\pi}{5}} =$$

$$= \frac{2\sin\frac{2\pi}{5}\cos\frac{2\pi}{5}}{4\sin\frac{\pi}{5}} = \frac{\sin\frac{4\pi}{5}}{4\sin\frac{\pi}{5}} = \frac{\sin\left(\pi-\frac{\pi}{5}\right)}{4\sin\frac{\pi}{5}} = \frac{\cancel{\sin\frac{\pi}{5}}}{4\cancel{\sin\frac{\pi}{5}}} = \frac{1}{4} = R.S.$$

Example 2:
Simplify:

(a) $5\sin\frac{\pi}{12}\cos\frac{\pi}{12} =$

$$= \frac{5}{2}\left(2\sin\frac{\pi}{12}\cos\frac{\pi}{12}\right)$$

$$= \frac{5}{2}\left(\sin\frac{\pi}{6}\right) = \frac{5}{2}\left(\frac{1}{2}\right)$$

$$= \frac{5}{4}$$

(b) $4\cos(-15^\circ)\sin(-15^\circ)$

$$= 2\left(2\cos(-15^\circ)\sin(-15^\circ)\right)$$

$$= 2\sin(-30^\circ)$$

$$= -2\sin 30^\circ$$

$$= -2\left(\frac{1}{2}\right) = -1$$

Example 3:

If $\tan x = \frac{4}{3}$, $\pi < x < \frac{3\pi}{2}$ (Q III), find the value of $\tan\frac{x}{2}$

$\tan x = \frac{y}{x}$, $x^2+y^2=r^2$

$\begin{cases} x=3 \\ y=4 \end{cases}$ ✓

$3^2+4^2=r^2$

$r^2=25$, $r=\pm 5$

$r>0$

$r=5$

$$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

$$\sin x = -\frac{4}{5}$$

$$\cos x = -\frac{3}{5}$$

$$\tan\left(\frac{x}{2}\right) = \frac{-\frac{4}{5}}{1-\frac{3}{5}} = \frac{-\frac{4}{5}}{\frac{2}{5}} = -\frac{4}{\cancel{5}}\times\frac{\cancel{5}}{2} = -\frac{4}{2} = -2.$$

Answer: $\tan\frac{x}{2} = -2$

Practice
Evaluate (without using a calculator):

(a) $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$

(b) $\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}$

Example 4:

If $\sin x = \frac{4}{5}$, $\frac{\pi}{2} < x < \pi$, find the value of $\sin 2x$ QII

$\sin 2x = 2\sin x \cos x$

We need the value of $\cos x$: $\sin^2 x + \cos^2 x = 1$

$\cos x = \pm\sqrt{1-\sin^2 x}$

CAST says $\cos x < 0$ in QII

$\cos x = -\sqrt{1-\left(\frac{4}{5}\right)^2} = -\sqrt{1-\frac{16}{25}} = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

$\sin 2x = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = -\frac{24}{25}$

Answer: $\sin 2x = -\frac{24}{25}$ ✓

Example 4:

Evaluate (without a calculator) $\sin\frac{\pi}{8}$

$\sin\frac{\pi}{8} = \sin\left[\frac{1}{2}\left(\frac{\pi}{4}\right)\right] = \pm\sqrt{\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}}$

CAST says $\sin\frac{\pi}{8} > 0$

$\sin\frac{\pi}{8} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$

divide numerator and denominator of the radicand by $\sqrt{2}$

Answer: $\sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$