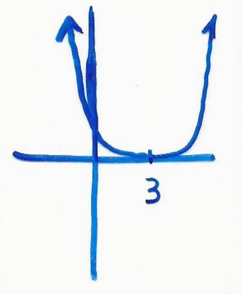
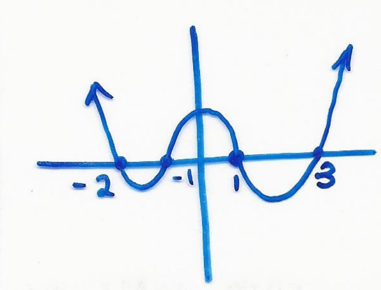
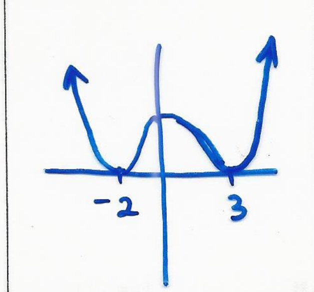
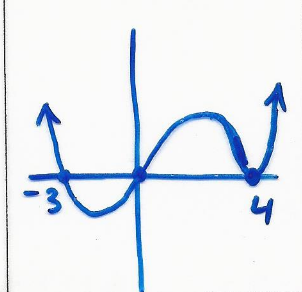
Equations and Graphs of Polynomial Functions

Date: ____________

Quartic Function standard form: $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

0, 1, 2, 3 or 4 real roots factored form: $f(x) = k(x-s)(x-t)(x-u)(x-v)$

Function	$y=(x-3)^4$ is obtained if $y=x^4$ is shifted	$y=(x+2)(x+1)(x-1)(x-3)$ x-intercepts: $-2, -1, 1, 3$ $a=1>0$	$y=(x+2)^2(x-3)^2$ x-intercepts: $x=-2, x=3$	$y=x(x+3)(x-4)^2$ $y=(x-0)(x+3)(x-4)^2$ x-intercepts $x=0,-3,4$ ← order 2
Sketch				
number of x intercepts	1 x-intercept $x=3$	4 x-intercepts	2 x-intercepts $x=-2, 3$	3 x-intercepts $x=-3,0,4$
Type of the roots of the related equation, $f(x)=0$	$0=(x-3)^4$ $x=3$ ← root of order 4 4 real equal roots	$0=(x+2)(x+1)(x-1)(x-3)$ 4 real distinct roots: $-2,-1,1,3$	$0=(x+2)^2(x-3)^2$ $x=-2, x=-2$ $x=3, x=3$ 2 pairs of real equal (in pairs) roots	2 real and distinct: $0, -3$ 2 real and equal: $4, 4$

There are more possibilities for quartic functions that are not listed above. Consider the following:

$y = x^4 + 16$ ← no real x-intercepts, $y \geq 16$

$y=(x+2)(x+1)(x^2+1)$ ← $x=-2$, $x=-1$, $x=\pm i$

$y=(x+2)^2(x^2+2x+2)$ ← $x=-2$ (order of 2), $x^2+2x+2 \neq 0$, $x^2+2x+2 \geq 1$

$y=(x+2)^2(x^2-3)^2-50$ ← $y=(x+2)^2(x^2-3)^2$ moved 50 units down.

$y=-x^4+2x^3-11x^2-12x-46$ ← factor if possible!

In general, zeros of even order do not change the sign of the function (the function "bounces off" the x-axis instead of crossing)

and zeros of odd order do change the sign of the function. (function crosses the x-axis!)