#8 Prove that

$$\cos^6 x + \sin^6 x = 1 - 3\sin^2 x + 3\sin^4 x$$

$L.S. = \cos^6 x + \sin^6 x = \ldots$ sum of perfect cubes:

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$$L.S. = \cos^6 x + \sin^6 x$$

$$= (\cos^2 x)^3 + (\sin^2 x)^3$$

$$= \underbrace{(\cos^2 x + \sin^2 x)}_{=1}(\cos^4 x - \cos^2 x \sin^2 x + \sin^4 x)$$

$$= \cos^4 x - \cos^2 x \sin^2 x + \sin^4 x$$

$$= \underbrace{\cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x}_{PST} - 3\cos^2 x \sin^2 x$$

$$= (\cos^2 x + \sin^2 x)^2 - 3\cos^2 x \sin^2 x$$

$$= 1 - 3\cos^2 x \sin^2 x$$

$$= 1 - 3(1 - \sin^2 x)\sin^2 x$$

$$= 1 - 3(\sin^2 x - \sin^4 x)$$

$$= 1 - 3\sin^2 x + 3\sin^4 x$$

$$= R.S.$$

QED