

Textbook: p III #14

Question:

The distance, d in kilometers, travelled by a plane after t hours can be represented by $d(t) = -4t^3 + 40t^2 + 500t$, where $0 \leq t \leq 10$. How long does it take the plane to fly 4088 km?

Solution:

$$\text{Set } -4t^3 + 40t^2 + 500t = 4088$$

move all terms to one side!

$$-4t^3 + 40t^2 + 500t - 4088 = 0$$

Divide through

by -4 :

$$t^3 - 10t^2 - 125t + 1022 = 0$$

divisors of 1022: $\pm 2, \pm 7, \pm 14$, etc

$$\text{let } f(t) = t^3 - 10t^2 - 125t + 1022 = 0$$

$f(7) = 0$, $\therefore (t-7)$ is a factor of $f(t)$

Perform Polynomial Long Division.

$$\begin{array}{r} t^2 - 3t - 146 \\ (t-7) \overline{) t^3 - 10t^2 - 125t + 1022} \\ \underline{t^3 - 7t^2} \\ -3t^2 - 125t \\ \underline{-3t^2 + 21t} \\ -146t + 1022 \\ \underline{-146t + 1022} \\ 0 \end{array}$$

$$\rightarrow f(t) = (t-7)(t^2 - 3t - 146)$$

$$0 = (t-7)(t^2 - 3t - 146)$$

$$t-7=0 \text{ or } t^2 - 3t - 146 = 0$$

$$t=7 \text{ or } t = \frac{3 \pm \sqrt{9 - 4(1)(-146)}}{2}$$

$$t=7 \text{ or } t = \frac{3 + \sqrt{593}}{2} \doteq 13.676$$

discard due to restriction of $0 \leq t \leq 10$ in the question

$$\text{or } t = \frac{3 - \sqrt{593}}{2} \doteq -10.676$$

(discard as time cannot be negative!)

Answer: $t = 7$ hours.