

When a polynomial is divided by  $(x+2)$ , the remainder is  $-19$ . When the same polynomial is divided by  $(x-1)$ , the remainder is  $2$ . Determine the remainder when the polynomial is divided by  $(x-1)(x+2)$

Let  $f(x)$  represent the polynomial (viewed as a function)

$$\begin{cases} f(-2) = -19 \\ f(1) = 2 \end{cases}$$

$$f(x) = \underbrace{(x-1)(x+2)}_{\text{new } d(x)} \cdot Q(x) + R(x)$$

$$\text{Let } R(x) = mx + n$$

then

$$\begin{cases} f(-2) = R(-2) \quad (\text{the first term is zero!}) \\ f(1) = R(1) \end{cases}$$

$$\begin{cases} f(-2) = -2m + n = -19 \\ f(1) = m + n = 2 \end{cases} \quad \left. \vphantom{\begin{cases} f(-2) = -2m + n = -19 \\ f(1) = m + n = 2 \end{cases}} \right\} \text{We have a linear system.}$$

$$\begin{cases} -2m + n = -19 \quad (1) \\ m + n = 2 \quad (2) \end{cases}$$

$$\begin{aligned} (2) - (1): & -3m = -21 \\ & m = 7 \end{aligned}$$

Sub into (2)

$$7 + n = 2$$

$$n = 2 - 7$$

$$n = -5$$

$$\therefore R(x) = 7x - 5$$