Thinking Questions

2) The remainder when $f(x)=x^5-2x^4+ax^3-x^2+bx-2$ is divided by $x+1$ is $-7$. When $f(x)$ is divided by $x-2$, the remainder is 32. Determine the remainder when $f(x)$ is divided by $x-1$.

Solution

$$f(x)=x^5-2x^4+ax^3-x^2+bx-2$$

$$\begin{cases} f(-1)=-7 \\ f(2)=32 \end{cases}$$

$$f(-1)=(-1)^5-2(-1)^4+a(-1)^3-(-1)^2+b(-1)-2$$

$$f(-1)=-1-2-a-1-b-2=-7$$

$$-6-a-b=-7$$

$$\boxed{a+b=1} \quad ①$$

$$f(2)=(2)^5-2(2)^4+a(2)^3-(2)^2+b(2)-2$$

$$f(2)=\cancel{32}-\cancel{32}+8a-4+2b-2=32$$

$$8a+2b-6=32, \quad 8a+2b=38$$

$$\boxed{4a+b=19} \quad ②$$

$$\begin{cases} a+b=1 & ① \\ 4a+b=19 & ② \end{cases}$$

② − ①: $3a=18$

$$a=6$$

Sub $a=6$ into ①:

$$6+b=1$$

$$b=-5$$

$$\therefore f(x)=x^5-2x^4+6x^3-x^2-5x-2$$

$$f(1)=(1)^5-2(1)^4+6(1)^3-(1)^2-5(1)-2$$

$$f(1)=1-2+6-1-5-2=-3$$ ✓