

Thinking Questions

1) If m , n , and 1 are non-zero roots of the equation $x^3 - mx^2 + nx - 1 = 0$, then find the sum of the roots.

Since 1 is a root

$$(1)^3 - m(1)^2 + n(1) - 1 = 0,$$

$$1 - m + n - 1 = 0, \quad n - m = 0, \quad \boxed{n = m}$$

Since m is a root

$$m^3 - m^3 + nm - 1 = 0, \quad mn - 1 = 0$$

$$\boxed{mn = 1}$$

Case 1:

$$\begin{cases} n = m \\ mn = 1 \rightarrow m^2 = 1 \\ m = \sqrt{1} \end{cases}$$

↓

$$m = 1, \quad n = 1$$

$$x^3 - x^2 + x - 1 = 0$$

$$x^2(x-1) + 1(x-1) = 0$$

$$(x-1)(x^2+1) = 0$$

one real solution

which contradicts

the conditions of the problem:

m , n , and 1 being non-zero (real) roots

Case 2:

$$\begin{cases} m = n \\ mn = 1 \rightarrow m^2 = 1 \\ m = -\sqrt{1} \end{cases}$$

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$$m = -1, \quad n = -1$$

$$x^3 + x^2 - x - 1 = 0$$

$$x^2(x+1) - 1(x+1) = 0$$

$$(x+1)(x^2-1) = 0$$

$$(x+1)(x-1)(x+1) = 0$$

$$x+1=0, \quad x-1=0, \quad x+1=0$$

$$x = -1 \text{ or } x = 1, \quad x = -1$$

$$\begin{aligned} \text{Sum} &= -1 + 1 - 1 \\ &= -1 \checkmark \end{aligned}$$