# Solving Polynomial Equations

Date:

Recall: Quadratic Formula, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$. Any equation of the form $f(x)=0$ can be solved if $f(x)$ can be expressed as a combination of linear and quadratic factors.

**Example 1**

a) Find the family of cubic functions whose x-intercepts are –3, 0, and 2.

b) Find the particular member of the above family whose graph passes through the point (-1, 12).

(a) $f(x)=a(x-(-3))(x-0)(x-2)$

$f(x)=ax(x+3)(x-2)$ ← this is a family of cubic functions which is a group of cubic functions sharing same characteristic(s) namely x-intercepts.

(b) Sub (-1, 12): $12=a(-1)(-1+3)(-1-2)$ → $f(x)=2x(x+3)(x-2)$

$12=6a,\ a=2$

**Example 2**

Solve $3x^3+x^2-12x-4=0$

Grouping works nicely here: $\underbrace{3x^3+x^2}_{\text{group 1}}\underbrace{-12x-4}_{\text{group 2}}=0$

$x^2(3x+1)-4(3x+1)=0$

$(3x+1)(x^2-4)=0$

$(3x+1)(x-2)(x+2)=0$

ZPP: $3x+1=0$ or $x-2=0$ or $x+2=0$

$x=-\frac{1}{3}$ or $x=2$ or $x=-2$

**Example 3**

Solve a) $x^3+9x^2+13x+5=0$

divisors of 5: $\pm1, \pm5$

let $f(x)=x^3+9x^2+13x+5$

$f(-1)=(-1)^3+9(-1)^2+13(-1)+5$

$f(-1)=0 \Rightarrow (x+1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2+8x+5 \\ x+1 & x^3+9x^2+13x+5 \\ & x^3+x^2 \\ \hline & 8x^2+13x \\ & 8x^2+8x \\ \hline & 5x+5 \\ & 5x+5 \\ \hline & 0 \end{array}$$

$\therefore f(x)=(x+1)(x^2+8x+5)=0$

$x+1=0$ or $x^2+8x+5=0$

$x=-1$ or

→ $x^2+8x+5=0$

$x_{1,2}=\dfrac{-8\pm\sqrt{8^2-4(1)(5)}}{2(1)}$

$x_{1,2}=\dfrac{-8\pm\sqrt{44}}{2}$

$x_{1,2}=\dfrac{-8\pm2\sqrt{11}}{2}$

$x_{1,2}=-4\pm\sqrt{11}$

Answer: $-1$, $-4+\sqrt{11}$, $-4-\sqrt{11}$

b) $x^3+4x-5=0$

let $f(x)=x^3+4x-5$

$f(1)=0$, $\therefore (x-1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2+x+5 \\ x-1 & x^3+0x^2+4x-5 \\ & x^3-x^2 \\ \hline & x^2+4x \\ & x^2-x \\ \hline & 5x-5 \\ & 5x-5 \\ \hline & 0 \end{array}$$

$x^3+4x-5=(x-1)(x^2+x+5)=0$

$x-1=0$ $\boxed{\text{OR}}$ $x^2+x+5=0$

$x=1$

$D=(1)^2-4(1)(5)=-19<0$ no real solutions

Answer: $x=1$ (and two complex conjugate solutions)

$6x^2 - x - 1$ $\quad \left\{ \begin{matrix} p = -6 \\ s = -1 \end{matrix} \right\} \Rightarrow -3, 2$

$= 6x^2 - 3x + 2x - 1$

$= 3x(2x-1) + 1(2x-1)$

$= (3x+1)(2x-1)$



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**Example 4**

Solve $6x^3 - 13x^2 + x + 2 = 0$

divisors of 2: $\pm 1, \pm 2$

divisors of 6: $\pm 1, \pm 2, \pm 3, \pm 6$

Testing Values: $\pm 1, \pm 2, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{1}{6}$

let $f(x) = 6x^3 - 13x^2 + x + 2$

$f(2) = 6(2)^3 - 13(2)^2 + 2 + 2 = 0 \Rightarrow (x-2)$ is a factor of $f(x)$ ← therefore

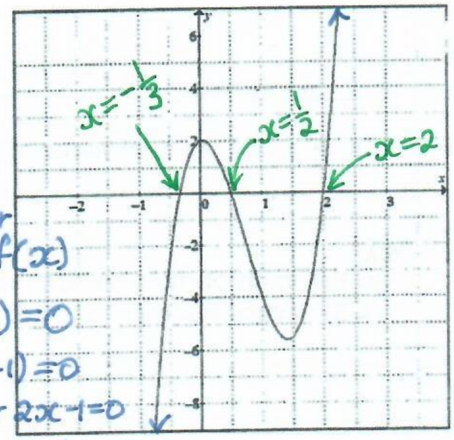
$$\begin{array}{r} 6x^2 - x - 1 \\ x-2 \overline{)\, 6x^3 - 13x^2 + x + 2} \\ \underline{6x^3 - 12x^2} \qquad\qquad \\ -x^2 + x \qquad \\ \underline{-x^2 + 2x} \qquad \\ -x + 2 \\ \underline{-x + 2} \\ 0 \end{array}$$

$f(x) = (x-2)(6x^2 - x - 1) = 0$

$f(x) = (x-2)(3x+1)(2x-1) = 0$

$x - 2 = 0$ or $3x + 1 = 0$ or $2x - 1 = 0$

$x = 2$ or $x = -\frac{1}{3}$ or $x = \frac{1}{2}$




**Example 5**

Solve $x^4 - 24x^2 - 25 = 0$

With a suitable substitution (replacement) we can see that it is a quadratic in nature (or quadratic in form)

Indeed: let $x^2 = a$, $a \geq 0$ (Any perfect square of a real number is non-negative)

$a^2 - 24a - 25 = 0$

$(a - 25)(a + 1) = 0$

$(x^2 - 25)(x^2 + 1) = 0$

$(x-5)(x+5)(x^2+1) = 0$

$x - 5 = 0$ or $x + 5 = 0$ or $x^2 + 1 = 0$

$x = 5$ or $x = -5$

this equation has no real solutions

$x^2 = -1$

$x = \pm\sqrt{-1}$, $\sqrt{-1} = i$

$x = \pm i$

Remark: 2 real roots + 2 imaginary (conjugate) roots give us 4 roots altogether which is the degree of $f(x) = x^4 - 24x^2 - 25$

**Example 6**

Solve $(x^2 - 5x - 5)(x^2 - 5x + 3) = 9$

It is a good strategy to replace an expression that $\boxed{\text{repeats}}$ exactly with another variable.

let $x^2 - 5x = k$

$(k-5)(k+3) = 9$

$k^2 + 3k - 5k - 15 - 9 = 0$

$k^2 - 2k - 24 = 0$

$(k-6)(k+4) = 0$

$k - 6 = 0$ or $k + 4 = 0$

$k = 6$ or $k = -4$

Replace $k$ with what it stands for:

$x^2 - 5x = 6$ or $x^2 - 5x = -4$

$x^2 - 5x - 6 = 0$ | $x^2 - 5x + 4 = 0$

$(x-6)(x+1) = 0$ | $(x-4)(x-1) = 0$

$x - 6 = 0$ or $x + 1 = 0$ | $x - 4 = 0$ or $x - 1 = 0$

$x = 6$ or $x = -1$ | $x = 4$ or $x = 1$

**Homework**

~~P.110 #3 – 5, 6 – 8 eo, 14, 15, 18, 20, 22~~

Please See the website for HW!



The four (quartic!) solutions are:

$x_1 = 6$, $x_2 = -1$, $x_3 = 4$, $x_4 = 1$.