

The Remainder Theorem says:

1. When a polynomial,  $f(x)$ , is divided by  $x - a$ , the remainder is equal to  $f(a)$ .
2. When a polynomial,  $f(x)$ , is divided by  $ax - b$ , the remainder is equal to  $f\left(\frac{b}{a}\right)$ .

Example 1:

Determine the remainder when  $x^3 + 7x^2 + 2x - 5$  is divided by  $x + 7$ .

Define a function: let  $f(x) = x^3 + 7x^2 + 2x - 5$

$$f(x) = (x + 7)Q(x) + R$$

$$R = f(-7)$$

$$R = (-7)^3 + 7(-7)^2 + 2(-7) - 5$$

$$\rightarrow R = -19$$

Example 2:

For what value of  $b$  will the polynomial  $f(x) = -2x^3 + bx^2 - 5x + 2$  have the same remainder when divided by  $x - 2$  and  $x + 1$ ?

$$R_1 = f(2), \quad R_2 = f(-1)$$

$f(2) = f(-1)$  since remainders are equal

$$f(2) = -2(2)^3 + b(2)^2 - 5(2) + 2 \rightarrow f(2) = -16 + 4b - 8 = -24 + b$$

$$f(-1) = -2(-1)^3 + b + 5 + 2 \rightarrow f(-1) = b + 9$$

$\uparrow$   
 $-5(-1)$

$$4b - 24 = b + 9$$

$$4b - b = 24 + 9, \quad 3b = 33, \quad b = 11$$

Example 3:

When  $x^3 + 3x^2 - kx + 10$  is divided by  $x - 5$ , the remainder is 15. Find the value of  $k$ .

$$\text{let } f(x) = x^3 + 3x^2 - kx + 10$$

$$\begin{cases} R = f(5) = (5)^3 + 3(5)^2 - k(5) + 10 \\ R = 15 \end{cases}$$

$$210 - 5k = 15$$

$$5k = 210 - 15$$

$$5k = 195$$

$$k = 39 \checkmark$$

Example 4:

When  $2x^3 - mx^2 + nx - 2$  is divided by  $x+1$ , the remainder is  $-12$ . The binomial  $x-2$  is a factor.

Determine the values of  $m$  and  $n$ .

let  $f(x) = 2x^3 - mx^2 + nx - 2$

① If divided by  $(x+1)$ ,

$$R = -12$$

$$f(-1) = -12$$

$$f(-1) = 2(-1)^3 - m(-1)^2 + n(-1) - 2 = -12$$

$$-m - n - 4 = -12$$

$$-m - n = -12 + 4$$

$$\boxed{m + n = 8}$$

② If divided by  $x-2$ ,  $R=0$  ("is a factor")

$$f(2) = 0$$

$$2(2)^3 - m(2)^2 + n(2) - 2 = 0$$

$$16 - 4m + 2n - 2 = 0$$

$$2n - 4m = -14$$

$$\boxed{n - 2m = -7}$$

We now solve a linear system:

$$\begin{cases} n + m = 8 & \longrightarrow 2m + 2n = 16 & \leftarrow \text{add equations} \\ n - 2m = -7 & \longrightarrow \underline{n - 2m = -7} & \leftarrow \text{to eliminate } m. \end{cases}$$
$$3n = 9$$
$$n = 3$$

then  $3 + m = 8$

$$m = 8 - 3$$

$$m = 5$$

Answer:  $m=5, n=3$

Answers:

1.  $R = -19$

2.  $b = 11$

3.  $k = 39$

4.  $m = 5$  and  $n = 3$