Rational Function

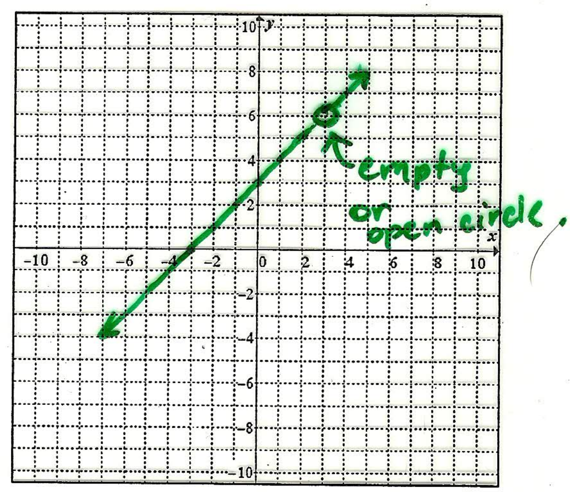
A rational function is a function of the form $f(x)=\dfrac{P(x)}{Q(x)}$, where P(x) and Q(x) are polynomial functions. The domain of f(x) excludes all x for which Q(x) = 0. In other words, a rational function of x is a quotient of two polynomial functions of x.

Point Discontinuities ("Holes")

If P(x) and Q(x) have the same zero, c, or in other words, have the same factor $x-c$, then the graph of $f(x)=\dfrac{P(x)}{Q(x)}$ has a point discontinuity (hole) (and not a vertical asymptote) at x = c.

Example:

$$y=\frac{x^2-9}{x-3}=\frac{(x-3)(x+3)}{x-3}=x+3,\ x\neq 3$$



The graph of $y=f(x)$ is the same as the graph of $y=x+3$, except that an open circle at x = 3 indicates that the graph of $y=\dfrac{x^2-9}{x-3}$ does not contain the point (3 , 6).

Asymptotes: An asymptote is a line that a graph approaches, but does not touch, as the curve of the graph is extended indefinitely. There may be horizontal, vertical, or slanted (oblique) asymptotes.

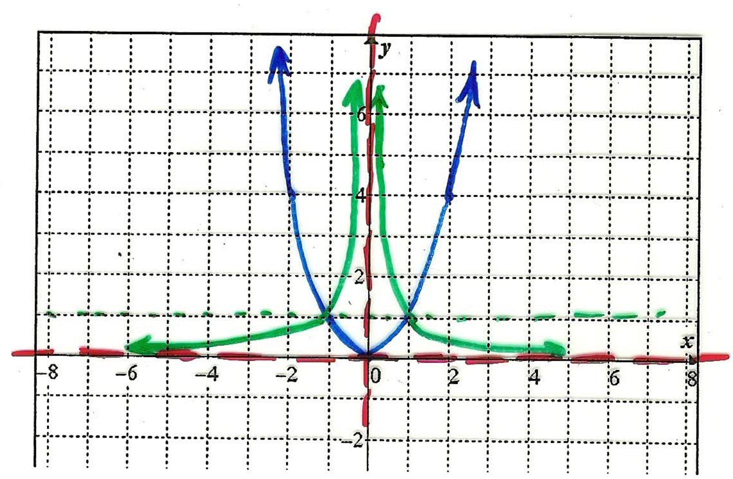
Vertical Asymptotes: x = a is a VA of the graph of $y=f(x)$ if as $x\to a$, $y\to\infty$ or $y\to-\infty$

Example: $y=\dfrac{1}{x^2}$

$x^2=0,\ x=0$ (y-axis)

even function as $f(-x)=\dfrac{1}{x^2}=f(x)$

graph is symmetric in y-axis



Horizontal Asymptotes:

A line y = b is a horizontal asymptote for the graph of $y=f(x)$ if $y\to b$ as $x\to\infty$ or $x\to-\infty$

Example: $y=\dfrac{1}{x^2}$, $y\neq 0$ HA: $y=0$

but $y\to 0$ as $x\to\infty$

and

as $x\to-\infty$