MHF4U

Estimate the Instantaneous Rate of Change (IRC) – Part 2

Example1:

Determine the instantaneous rate of change of $y=-x^2+6x$ at $x=5$.

Method 1: Using Preceding Interval

Determine the average rate of change over shorter and shorter intervals.

Interval	Δy	Δx	$\frac{\Delta y}{\Delta x}$
$f(5)=-(5)^2+6(5)=5$ $f(4.5)=6.75$ $4.5\le x\le 5$	$\Delta y=y_{x=5}-y_{x=4.5}$ $\Delta y=f(5)-f(4.5)$ $\Delta y=5-6.75$ $\Delta y=-1.75$	$\Delta x=5-4.5$ $\Delta x=0.5$	$\frac{\Delta y}{\Delta x}=\frac{-1.75}{0.5}$ $\frac{\Delta y}{\Delta x}=-3.5$
$f(5)=5$ $4.9\le x\le 5$ $f(4.9)=5.39$	$\Delta y=f(5)-f(4.9)$ $\Delta y=5-5.39$ $\Delta y=-0.39$	$\Delta x=5-4.9$ $\Delta x=0.1$	$\frac{\Delta y}{\Delta x}=\frac{-0.39}{0.1}$ $\frac{\Delta y}{\Delta x}=-3.9$
$4.99\le x\le 5$ $f(4.99)=5.0399$	$\Delta y=f(5)-f(4.99)$ $\Delta y=5-5.0399$ $\Delta y=-0.0399$	$\Delta x=5-4.99$ $\Delta x=0.01$	$\frac{\Delta y}{\Delta x}=\frac{-0.0399}{0.01}$ $\frac{\Delta y}{\Delta x}=-3.99$

The sequence of values of ARC is: $-3.5, -3.9, -3.99$ so $IRC_{(x=5)} \doteq -4$

Method 2: Using the difference quotient (the following interval)

$$ARC=\frac{f(a+h)-f(a)}{h}=\frac{f(5+h)-f(5)}{h}=\frac{[-(5+h)^2+6(5+h)]-5}{h}$$

$$ARC=\frac{-(25+10h+h^2)+30+6h-5}{h}$$

$$ARC=\frac{-25-10h-h^2+30+6h-5}{h}=\frac{-4h-h^2}{h}=-4-h$$

Choose $h=0.001$

Then $IRC \doteq -4-0.001$

$IRC \doteq -4.001$ or $IRC \doteq -4$

Example 2:

A population of raccoons moves into a wooded area. At t months, the number of raccoons, $P(t)$, can be modelled by the function $P(t) = 100 + 30t + 4t^2$.

a) Determine the population of racoons at 2.5 months.

$P(2.5) = 100 + 30(2.5) + 4(2.5)^2$

$P(2.5) = 100 + 75 + 25 = 200$ raccoons

b) Determine the average rate of change in the raccoon population over the interval from 0 to 2.5 months.

$P(2.5) = 200, \quad P(0) = 100. \quad ARC = \frac{\Delta P}{\Delta t} = \frac{P(2.5) - P(0)}{2.5 - 0}$

$ARC = \frac{\Delta P}{\Delta t} = \frac{200 - 100}{2.5 - 0} = \frac{100}{2.5} = 40$ raccoons/month

c) Estimate the rate of change in the raccoon population at exactly 2.5 months. → IRC

Use DQ

$DQ = \frac{P(2.5 + h) - P(2.5)}{h} = \frac{100 + 30(2.5 + h) + 4(2.5 + h)^2 - 200}{h}$

d) explain why your answers for parts a), b), and c) are all different.

$DQ = \frac{100 + 75 + 30h + 4(6.25 + 5h + h^2) - 200}{h}$

$DQ = \frac{100 + 75 + 30h + 25 + 20h + 4h^2 - 200}{h} = \frac{50h + 4h^2}{h} = 50 + 4h$

$IRC \doteq 50 + 4(0.001) = 50 + 0.004 = 50.004$ or $IRC \doteq 50.004$ rac/month

Example 3:

A diver is on a 10 m platform, preparing to dive. The diver's height above the water, in meters, at time t can be modelled using the equation $h(t) = 10 - 4.9t^2$

a) Determine when the diver will enter the water.

b) Estimate the rate at which the diver's height above the water is changing as the diver enters the water.

a) Diver enters the water when $h(t) = 0$

$0 = 10 - 4.9t^2, \quad 10 = 4.9t^2, \quad \frac{10}{4.9} = t^2, \quad t^2 \doteq 2.0408$

$t \doteq \sqrt{2.0408} \doteq 1.43$

b) $ARC = \frac{h(1.43 + h) - h(1.43)}{h} = \frac{[10 - 4.9(1.43 + h)^2] - 0.59346}{h}$

$ARC = \frac{10 - 7.007 - 4.9h - 0.59346}{h} \doteq 2.4 - 4$

Answers:

Example 2: a) 200 raccoons b) +40 raccoons/month