The volume of a cubic crystal can be modelled by $V(x)=x^3$, where $V(x)$ is the volume measured in cubic centimeters and x is the side length in cm. Estimate the instantaneous rate of change of $V(x)$ when $x=5$.

method 1: Centered Interval Approach.

$5-h \leq x \leq 5+h$, h is a small number.

$h=0.5$

$4.5 \leq x \leq 5.5$

$$ARC_{[4.5,5.5]} = \frac{V(5.5)-V(4.5)}{5.5-4.5}$$

$$ARC \doteq \frac{166.375-91.125}{5.5-4.5}$$

$$ARC = \frac{\Delta V}{\Delta X} \doteq 75.25 \frac{cm^3}{cm}$$

do not reduce!

$h=0.1$

$4.9 \leq x \leq 5.1$

$$ARC_{[4.9,5.1]} = \frac{V(5.1)-V(4.9)}{5.1-4.9}$$

$$ARC = \frac{\Delta V}{\Delta X} = \frac{132.651-117.649}{5.1-4.9}$$

$$ARC \doteq 75.01 \frac{cm^3}{cm}$$

$h=0.01$

$4.99 \leq x \leq 5.01$

$$ARC_{[4.99,5.01]} = \frac{\Delta V}{\Delta X} = \frac{125.751501-124.251499}{5.01-4.99}$$

$$\Rightarrow \frac{\Delta V}{\Delta X} \doteq 75.0001 \frac{cm^3}{cm}$$

$$IRC \doteq 75 \frac{cm^3}{cm}.$$

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method 2: Using Difference Quotient

$$D.Q. = \frac{f(a+h)-f(a)}{h}, \text{ a is the "point" of interest.}$$

$a=5$

$IRC \doteq \ldots$

$$\frac{f(5+h)-f(5)}{h} = \frac{(5+h)^3-5^3}{h} = \frac{(5+h)^3-125}{h}$$

$$= \frac{(5+h)(25+10h+h^2)-125}{h} = \frac{\cancel{125}+50h+5h^2+25h+10h^2+h^3-\cancel{125}}{h}$$

$$= \frac{h^3+15h^2+75h}{h} = \frac{\cancel{h}(h^2+15h+75)}{\cancel{h}} = h^2+15h+75$$

Choose $h=0.0001$ ✓ $\boxed{\boxed{h \neq 0}}$

$$IRC \doteq (0.0001)^2+15(0.0001)+75$$

$$IRC \doteq 75 \frac{cm^3}{cm}.$$