Factor Theorem – Part 2 ($a \neq 1$)

> Factor Theorem:
> If (and only if) k is substituted for x in f(x), and f(k) = 0 , then $x-k$ is a factor of f(x).
> Also if an only if $kx-b$ is a factor of P(x), then $P\left(\frac{b}{k}\right)=0$.

Example:
Is $x-2$ a factor of $P(x)=x^3+2x^2-10x+4$?

$k=2$

$P(2)=(2)^3+2(2)^2-10(2)+4=8+8-20+4=0$

$\therefore x-2$ is a factor of $P(x)$.

If the leading coefficient of the original polynomial is not 1, the $\underbrace{\textbf{rational}}_{\text{fractional}}$ **zero theorem** states:

If P(x) is a polynomial function with integer coefficients and $x=\frac{b}{k}$ is a rational zero of P(x), then

- b is a factor of the constant term of P(x)
- k is the factor of the leading coefficient of P(x)
- $kx-b$ is a factor of P(x)

Example:
Is $3x-2$ a factor of $T(x)=9x^3+2x-4$?

Set $3x-2=0$, $x=\frac{2}{3}$

$T(\frac{2}{3})=9(\frac{2}{3})^3+2(\frac{2}{3})-4=9(\frac{8}{27})+\frac{4}{3}-4$

$T(\frac{2}{3})=\frac{8}{3}+\frac{4}{3}-\frac{12}{3}=0$, $\therefore (3x-2)$ is a factor of $T(x)$

Example:
Factor $2x^3-3x^2-5x+6$

We need to find a value of x, such that $P(x)=0$

We use RZT (rational zero theorem)

factors of b: $(b=6)$ $\pm1, \pm2, \pm3, \pm6$

factors of k: $(k=2)$ $\pm1, \pm2$

Testing values: $\pm1, \pm2, \pm3, \pm6, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm3$

$P(1)=2(1)^3-3(1)^2-5(1)+6=2-3-5+6=0$

$\therefore x-1$ is a factor of $P(x)$

$$
\begin{array}{r|l}
 & 2x^2-x-6 \\ \hline
x-1 & 2x^3-3x^2-5x+6 \\
 & \underline{2x^3-2x^2} \\
 & -x^2-5x \\
 & \underline{-x^2+x} \\
 & -6x+6 \\
 & \underline{-6x+6} \\
 & 0
\end{array}
$$

$\therefore 2x^3-3x^2-5x+6=(x-1)(2x^2-x-6)$

$=(x-1)(x-2)(2x+3)$

Aside: $p=-12$, $s=-1$

$2x^2-x-6$

$=\underbrace{2x^2-4x}+\underbrace{3x-6}$

$=2x(x-2)+3(x-2)$

$=(x-2)(2x+3)$