MHF

# Factor Theorem ($a=1$) ← the leading coefficient of $P(x)$ is 1.

The Remainder Theorem says:

**When a polynomial, $f(x)$, is divided by $x-a$, the remainder is equal to $f(a)$.**

If this remainder is zero, then $x-a$ is a factor of f(x).

This constitutes a special case of the Remainder Theorem and is called the Factor Theorem.

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| Factor Theorem:<br>1. If (and only if) a is substituted for x in f(x), and f(a) = 0 , then $x-a$ is a factor of f(x). |

Example:

Determine which binomials are factors of $x^3-x^2-10x-8$. Let $f(x)=x^3-x^2-10x-8$.

a) $x+2$, $a=-2$

$f(-2)$
$=(-2)^3-(-2)^2-10(-2)-8$
$=-8-4+20-8$
$=0$
$\therefore R=0 \rightarrow$ $x+2$ is a factor of $f(x)$

b) $x-1$, $a=1$

$f(1)=(1)^3-(1)^2-10(1)-8$
$=-18\neq 0$
$\therefore$ $x-1$ is not a factor of $f(x)$

c) $x+1$, $a=-1$

$f(-1)=(-1)^3-(-1)^2-10(-1)-8$
$=-1-1+10-8$
$=10-10$
$=0$
$\therefore$ $x+1$ is a factor of $f(x)$

d) $x-4$, $a=4$

$f(4)=(4)^3-(4)^2-10(4)-8$
$f(4)=64-16-40-8$
$f(4)=64-64$
$f(4)=0$
$\therefore$ $x-4$ is a factor of $f(x)$

We found the three factors of $x^3-x^2-10x-8$. They are: $(x+2)$, $(x+1)$, $(x-4)$.

The product of these factors is the polynomial itself.

Therefore,

$x^3-x^2-10x-8=(x+2)(x+1)(x-4)$

$\uparrow$ constant term

The product of the constant terms in the factors is ( + 2)(+1)(-4) = - 8

This is the constant term in the original polynomial.

We can see that the zeroes go into the constant term of the original polynomial.

zeroes: $-2, -1, 4$ They all go into $-8$

This is summarized in the **Integral Zero Theorem** (Factor Property):

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| If $x-b$ is a factor of a polynomial function P(x) with the leading coefficient of 1 and remaining coefficients that are integers, then b is a factor of the constant term of P(x). |

We can use factor theorem and the integral zero theorem to factor a polynomial. Values of x that turn P(x) into zero need to be found. The integral zero theorem helps us figure out which values to test.

Example: Let $f(x)=x^3+3x^2-6x-8$

Factor $x^3+3x^2-6x-8$, $-1, -2, -4, -8, 1, 2, 4, 8$

$f(-1)=0$, $\therefore$ $x+1$ is a factor of $f(x)$. Use polynomial division.

$$
\begin{array}{r}
x^2+2x-8 \\
x+1\overline{)\,x^3+3x^2-6x-8} \\
\underline{x^3+x^2} \quad\quad\quad \\
2x^2-6x \quad \\
\underline{2x^2+2x} \quad \\
-8x-8 \\
\underline{-8x-8} \\
0
\end{array}
$$

$f(x)=(x+1)(x^2+2x-8)$

$f(x)=(x+1)(x+4)(x-2)$