Characteristics of Power Functions Date:

Investigation of Finite Differences

Investigation #2 (from text p. 17)

1. What is true about the third differences of a cubic function? the fourth differences of a quartic function?

They are the same or (non-zero) constant.

2. On a separate piece of paper, construct a finite difference table for each of the functions below. Use x-values from -3 to 4.

$y = x^3$ $y = -2x^3$ $y = x^4$ $y = -2x^4$

3. How is the sign of the leading coefficient related to the sign of the constant value of the finite differences?

They are the same.

4. How is the value of the leading coefficient related to the constant value of the finite differences?

n^{th} difference $= a_n \cdot n!$ for $\boxed{f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0}$

$n! = 1 \times 2 \times 3 \times \dots \times (n-1) \times n$ or $n! = n(n-1)(n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$

Example: for a cubic function, 3rd diff $= a(6)$

5. Make a conjecture about the relationship between the constant finite differences and:

a) the degree of the polynomial → the number of constant finite difference

b) the sign of the leading coefficient is the same as the sign of the constant finite difference

c) the value of the leading coefficient

$a = \dfrac{n\text{th diff}}{n!}$ ← from $\boxed{n_{th}\text{ diff} = an!}$

6. Verify your conjectures by constructing a finite difference table for $y = 3x^3 - 4x^2 + 1$ → and $y = -2x^4 + x^3 + 3x - 1$. Complete this work on a separate piece of paper.

$n = 3$

$\underbrace{n_{th}}_{\text{3rd diff}}$ diff $= 3(3!) = (3)(6) = 18$

→ 4_{th} const diff $= (-2)(4!) = -2(24) = -48$

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SUMMARY:
If we know that the n^{th} differences of a polynomial function are constant, we can conclude:

- The degree of the function is n
- The sign of the n^{th} differences is the same as the sign of the leading coefficient
- The value of the n^{th} differences is equal to $a[n(n-1)(n-2)...(3)(2)(1)] = an!$, where a is the leading coefficient

**This means that by calculating the finite differences, we can determine the leading(first) term of a polynomial function.

Examples:
1. What is the value of the fourth differences for the function $y = -2x^4$?

4rth diff $= -2(4!) = -2(24) = -48$

2. If the 5^{th} differences of a polynomial function are – 60, what is the leading term of the polynomial?

$a = \frac{-60}{5!} = \frac{-60}{120} = -\frac{1}{2}$

leading term: $-\frac{1}{2}x^5$

HW: p. 26 #3ii, iii, 7, 8adef, 11 – 18