p. 541 #8

(a) Determine the coordinates of the point on the line $\vec{r}=(1,-1,2)+s(1,3,-1)$, $s\in\mathbb{R}$ that produces the shortest distance between the line and a point with coordinates $(2,1,3)$

(b) What is the distance between the given point and the line?

Solution: $\vec{r}=(1,-1,2)+s(1,3,-1)$

(a) $\begin{cases} x=1+s \\ y=-1+3s \\ z=2-s \end{cases}$ Any point on the line is of the form $Q(1+s,-1+3s,2-s)$

We construct a vector from $P(2,1,3)$ to a general point Q on the line.

$\vec{PQ}=(2-1-s,\ 1+1-3s,\ 3-2+s)=(1-s,2-3s,1+s)$

The shortest distance is perpendicular distance

$\therefore\ \vec{PQ}\perp\vec{d}$, where $\vec{d}=(1,3,-1)$

$\therefore\ \vec{PQ}\cdot\vec{d}=0$

or

$(1,3,-1)\cdot(1-s,2-3s,1+s)=0$

$(1-s)+(6-9s)-(1+s)=0$

$1-s+6-9s-1-s=0,\quad 6-11s=0,\quad s=\frac{6}{11}$

This means the minimal distance will occur between $P(2,1,3)$ and the line when $s=\frac{6}{11}$

This point corresponds to

$(x,y,z)=(1,-1,2)+\frac{6}{11}(1,3,-1)=(1,-1,2)+(\frac{6}{11},\frac{18}{11},\frac{-6}{11})$

$(x,y,z)=(\frac{17}{11},\frac{7}{11},\frac{16}{11})$

(b) We need to find distance between $(\frac{17}{11},\frac{7}{11},\frac{16}{11})$ and $(2,1,3)$

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}=$

$=\sqrt{(2-\frac{17}{11})^2+(1-\frac{7}{11})^2+(3-\frac{16}{11})^2}=\sqrt{(\frac{5}{11})^2+(\frac{4}{11})^2+(\frac{17}{11})^2}=\sqrt{\frac{25}{121}+\frac{16}{121}+\frac{289}{121}}$

$=\sqrt{\frac{230}{121}}\doteq\sqrt{2.7272}\doteq 1.65$