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The point $A(2,4,-5)$ is reflected in the line with equation $\vec{r}=(0,0,1)+s(4,2,1)$, $s\in\mathbb{R}$, to give the point A'. Determine the coordinates of A'.

The line $\vec{r}=(0,0,1)+s(4,2,1)$ has parametric equations:

$$\begin{cases} x=4s \\ y=2s \\ z=1+s \end{cases}$$

$s\in\mathbb{R}$, Then any point on the line is of the form $Q(4s,2s,1+s)$

Then $\vec{QA}=(2,4,-5)-(4s,2s,1+s)$

$\vec{QA}=(2-4s,4-2s,-6-s)$

If Q is at a minimal distance from A, then this vector will be perpendicular to the direction vector of the line, namely, $(4,2,1)$.

Then $(4,2,1)\cdot(2-4s,4-2s,-6-s)=0$

$8-16s+8-4s-6-s=0$ or $10-21s=0$

$s=\frac{10}{21}$

$\therefore$ The point Q on the line at a minimal distance from A is:

substitution is a step!

$Q(4s,2s,1+s)=Q\left(4\left(\frac{10}{21}\right),2\left(\frac{10}{21}\right),1+\left(\frac{10}{21}\right)\right)$

$Q=\left(\frac{40}{21},\frac{20}{21},\frac{31}{21}\right)$ Then $\vec{QA}=\left(2-\frac{40}{21},4-\frac{20}{21},-5-\frac{31}{21}\right)$

$\vec{QA}=\left(\frac{2}{21},\frac{64}{21},-\frac{136}{21}\right)$ and A' will satisfy $\vec{QA'}=-\vec{QA}$

$\vec{QA'}=\left(-\frac{2}{21},-\frac{64}{21},\frac{136}{21}\right)=\vec{OA'}(a,b,c)-\vec{OQ}$

$=\left(a-\frac{40}{21},b-\frac{20}{21},c-\frac{31}{21}\right)=\left(-\frac{2}{21},-\frac{64}{21},+\frac{136}{21}\right)$

$$\left.\begin{cases} a-\frac{40}{21}=-\frac{2}{21} \longrightarrow a=\frac{38}{21} \\ b-\frac{20}{21}=-\frac{64}{21} \longrightarrow b=-\frac{44}{21} \\ c-\frac{31}{21}=+\frac{136}{21} \longrightarrow c=\frac{167}{21} \end{cases}\right\} \longrightarrow A'\left(\frac{38}{21},\frac{-44}{21},\frac{167}{21}\right)$$