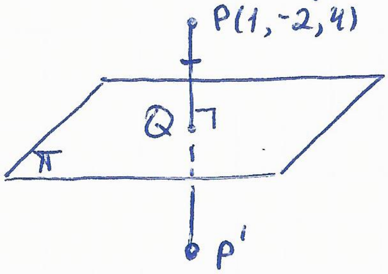
Vector Review – p. 558 # 14

If $P(1,-2,4)$ is reflected in the plane with equation $2x-3y-4z+66=0$, determine the coordinates of its image point, P'.
(Note that the plane $2x-3y-4z+66=0$ is the right bisector of the line joining $P(1,-2,4)$ with its image)



$\pi: 2x-3y-4z+66=0$

The plane is the right bisector of the line joining $P(1,-2,4)$ with its image.
The line connecting the two points, P and P', has a direction vector equal to that of the normal vector for the plane:
$\vec{n}=(2,-3,-4)$

So the line connecting the two points is:
$(x,y,z)=(1,-2,4)+t(2,-3,-4),\ t\in\mathbb{R}$

OR
$$\begin{cases} x=1+2t \\ y=-2-3t \\ z=4-4t \end{cases}$$

The intersection b/n the line and the plane, Q, is halfway between P and its image. We can find Q:

$2x-3y-4z+66=2(1+2t)-3(-2-3t)-4(4-4t)+66$
$=2+4t+6+9t-16+16t+66$
$=58+29t=0$
$t=-2$

Since Q is the midpoint of PP', image occurs at $t=2(-2)=-4$

P':
$$\begin{cases} x=1+2(-4)=-7 \\ y=-2-3(-4)=10 \\ z=4-4(-4)=20 \end{cases}$$

So $P'(-7,10,20)$

Alternatively:

Q:
$$\begin{cases} x=1+2(-2)=-3 \\ y=-2-3(-2)=4 \\ z=4-4(-2)=12 \end{cases}$$

Alternatively: $Q=\left(\frac{1+x}{2},\frac{-2+y}{2},\frac{4+z}{2}\right)=(-3,4,12)$

$\frac{1+x}{2}=-3 \Rightarrow x=-7$
$\frac{-2+y}{2}=4 \Rightarrow y=10$
$\frac{4+z}{2}=12 \Rightarrow z=20$