p. 533 #15

Consider the system of equations

① $4x + 3y + 3z = -8$

② $2x + y + z = -4$

③ $3x - 2y + (m^2 - 6)z = m - 4$

Determine the value of m for which this system of equations will have

(a) no solutions

(b) one solution

(c) an infinite number of solutions.

(a) We want to solve for z in terms of m so we can see different cases. This involves eliminating x and y.

$2x + y + z = -4$ ①′ switch ① and ②

$4x + 3y + 3z = -8$ ②′

③ × 2: $6x - 4y + (2m^2 - 12)z = 2m - 8$ ③′

③′ − 3①′: $\boxed{-7y + (2m^2 - 15)z = 2m + 4}$ **

②′ − 2·①′: $y + z = 0$ *

$2x + y + z = -4$

** + 7(*): $(2m^2 - 8)z = 2m + 4$. Now if $2m^2 - 8 = 0$

$m = \pm 2$

If $m = 2$, the equation becomes $0z = 8$ which has no solutions.

∴ No solution when $m = 2$.

(b) If $m \neq \pm 2$, $z = \dfrac{2m + 4}{2m^2 - 8}$ which is will have a unique solution for z.

Which, upon substitution in other equations gives unique solutions for x and y also.

There is a unique solution if $m \neq \pm 2$.

(c) Setting $m = -2$ we get $0z = 0$ which allows for z to be anything at all. So $m = -2$ will give an infinite number of solutions.