Given the line $(x,y,z)=(5,-2,-3)+t(9,3,-2)$, $t\in\mathbb{R}$ and the plane $3x-5y+7z=8$, find the point(s) of intersection.

Solution:

① Analysis: for line $\vec{d}=(9,3,-2)$

for plane $\vec{n}=(3,-5,7)$

$\vec{d}\cdot\vec{n}=(9,3,-2)\cdot(3,-5,7)=27-15-14=-2\neq 0$

$\therefore \vec{d}\not\perp\vec{n}$ and the line will intersect the plane at one point.

② To find the point:

line: $\begin{cases} x=5+9t \\ y=-2+3t \\ z=-3-2t \end{cases}$

Sub into equation of the plane and solve for t:

$3(5+9t)-5(-2+3t)+7(-3-2t)=8$

$15+27t+10-15t-21-14t=8$

$4-2t=8,\quad 2t=-4$

$t=-2$

Intersection is at

$(x,y,z)=(5,-2,-3)+(-2)(9,3,-2)$

$(x,y,z)=(-13,-8,1)$.