Example: Writing Equation(s) of a Line of Intersection of Two Planes.

Write the parametric and symmetric equations of the line of intersection of the planes
$2x - y + z = 5$ and $x + y - z = 1$

$\boxed{\text{Solution:}}$ π_1: $2x - y + z - 5 = 0$, π_2: $x + y - z - 1 = 0$

$\vec{n}_1 = (2, -1, 1)$; $\vec{n}_2 = (1, 1, -1)$.

We call the line of intersection of the planes ℓ.
let $\vec{d}$ the direction vector of the line.

$\vec{d} = \vec{n}_1 \times \vec{n}_2$

$$\left|\begin{array}{cccccc} 2 & -1 & 1 & 2 & -1 & 1 \\ 1 & 1 & -1 & 1 & 1 & -1 \end{array}\right|$$

$$\begin{array}{rrrr} & 1 & 1 & 2 \\ - & 1 & -2 & -1 \\ \hline & 0 & 3 & 3 \end{array} \Rightarrow \vec{d} = (0, 3, 3)$$

and $\vec{d}$ is parallel to line ℓ.
We only need a point on ℓ, and for that we solve the system

$$\begin{cases} 2x - y + z = 5 & (1) \\ x + y - z = 1 & (2) \end{cases}$$

From (2): $-y + z = x - 1$
Sub into (1)

$2x + x - 1 = 5$, $3x - 1 = 5$, $3x = 6$, $x = 2$

Substituting $x = 2$ does nothing...

let $z = t$, $y = 1 - 2 + t$, $y = -1 + t$

$$\begin{cases} x = 2 \\ y = -1 + t \\ z = t \end{cases}$$

Choose $t = 0$
$(2, -1, 0)$ is on the line

Symmetric ?

$x = 2, \quad y + 1 = z$ How is that for an attempt.